Physics 220 – Section 1 – Sample Midterm Test #2

Instructor: Lawrence Rees

CID: _______

Instructions

1. Be sure to write your CID on this paper.
2. Be sure your test has 26 questions.
2. Do not write your name on this test or we will not return it to you.
3. Enter your answer on the bubble sheet **and** in the box on the exam paper.
4. There are no time limits.
5. You may not use equations stored in your calculator.
6. Consider the number $A = 4.56 \times 10^{-7}$. The "exponent" of *A* is –7 and the first significant digit of *A* is 4. Do ***not*** round the first significant digit to 5!
7. All problems are worth 5 points each.
8. Unless stated otherwise, express all your answers in SI units.
9. If you feel a problem is in error, answer the best you can. I'll throw it out if there is no correct answer.

Possibly Useful Information:

$e = 1.602 \times 10^{-19}$ C

$k_e = 8.99 \times 10^{9}$ SI units

$\mu_0 = 4\pi \times 10^{-7}$ SI units

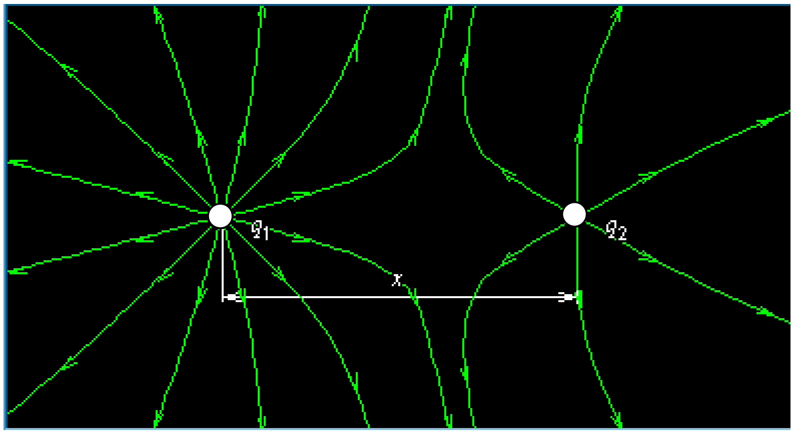
$\varepsilon_0 = 8.85 \times 10^{-12}$ SI units

G=10^9, M=10^6, k=10^3, m=10^{-3}, μ=10^{-6}, n=10^{-9}, p=10^{-12}

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

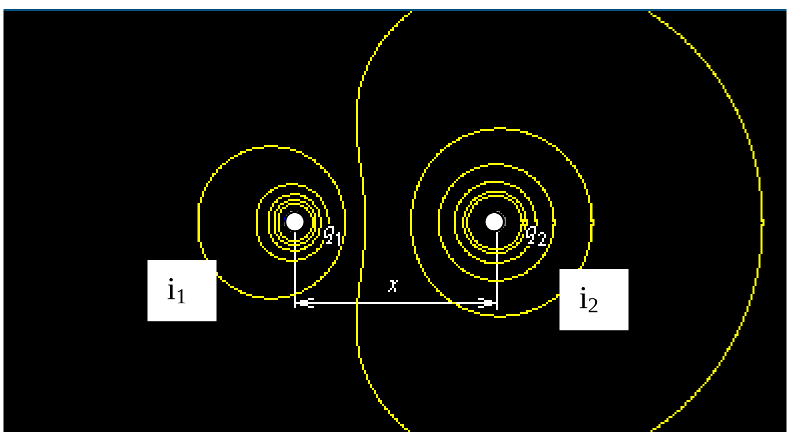
$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \hat{x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{y}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$



1. The figure above shows two charges. Which of the following is ***not*** true.

   A. The charge on the left is positive.
   B. 18 field lines go off to infinity.
   C. The charge on the left has twice the magnitude of the charge on the right.
   D. The two charges have opposite signs.
   E. What is shown is electric field lines.

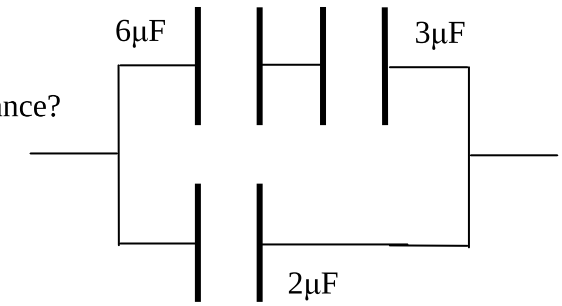


2. The diagram above illustrates the magnetic field of two current-carrying wires. The circles nearest the wires are not shown. Which of the following statements is ***not*** true?

   A. The current on the right is larger than the current on the left.
   B. The currents are traveling in opposite directions.
   C. What is shown are magnetic field lines.
   D. In the far field, we would see circles around both wires.
   E. There is a point between the wires where the net field is zero.

3. We draw a positive charge with 12 field lines and a negative charge with 8 field lines. What can we say of the field lines far from the charges?
   A. There are 4 field lines radiating outward.
   B. There are 4 field lines converging inward.
   C. There are 12 field lines radiating outward.
   D. There are 12 field lines converging inward.
   E. The field is too complicated to describe simply.

4. In the diagram to the right, what is the total capacitance?
   A. 1.00 μF
   B. 1.64 μF
   C. 2.50 μF
   D. 4.00 μF
   E. 11.0 μF
   F. None of the above



5. A battery is connected to a parallel plate capacitor that has air between the plates. The battery is then disconnected. A slab of dielectric is inserted between the plates. Which of the following happens?
   A. The voltage across the capacitor decreases.
   B. The total electric field between the plates increases.
   C. The charge on the capacitor increases.
   D. The capacitance remains constant.
   E. All of the above happen.

6. ^{14}C is a radioactive element that decays to ^{14}N by beta emission. The half-life of ^{14}C is 5730 years. This is time it takes for half of the ^{14}C to decay. Radioactive decay is mathematically similar to the discharge of a capacitor. Find the time constant for ^{14}C decay.
   A. 0.000175 years
   B. 0.000474 years
   C. 2108 years
   D. 8267 years
   E. 5730 years

7. A disk of radius R has a surface charge density of $\sigma = \alpha r^2$ where α is a constant. What is the total charge of the disk?

A. $q = \alpha\pi R^4 / 2$

B. $q = 2\alpha\pi R^3$

C. $q = \alpha\pi R^5 / 2$

D. $q = \alpha\pi r^5 / 2$

E. None of the above.

8. A sphere of radius R has a charge density of $\rho = \alpha r$ where α is a constant. What is the total charge of the sphere?

A. $q = 2\alpha\pi R^3 / 3$

B. $q = 2\alpha\pi r^3 / 3$

C. $q = \alpha\pi R^4$

D. $q = \alpha\pi r^4$

E. None of the above.

9. In a region of space, an electric potential is given by the expression $\alpha x^2 y - \alpha y^2 x$ where α is a constant. What is the electric field?

A. $-\alpha\left(x^2 - 2xy\right)\hat{x} - \alpha\left(2xy - y^2\right)$

B. $-\alpha\left(2xy - y^2\right)\hat{x} - \alpha\left(x^2 - 2xy\right)\hat{y}$

C. $\alpha\left(y^2 - x^2\right)$

D. 0

E. None of the above.

10. The definition of the curl is

A. $\lim_{\Delta a \to 0} \frac{\Phi}{\Delta a}$

B. $\lim_{\Delta v \to 0} \frac{\Phi}{\Delta v}$

C. $\lim_{\Delta a \to 0} \frac{\Lambda}{\Delta a}$

D. $\lim_{\Delta v \to 0} \frac{\Lambda}{\Delta v}$

E. None of the above.

11-12. A circuit consists of a capacitor of capacitance C=4.00 μF, a resistor of resistance R=12.0 Ω, and a switch all connected in series. The capacitor is charged to a voltage V_0 =2.00 V and at time $t=0$ the switch is closed so that the capacitor begins to discharge.

11. Find the time (in seconds) that it takes for the charge of the capacitor to drop to *1/e* of its initial value.

Mark the first significant digit of your answer.

12. What is the current at that time? Mark the first significant digit of your answer.

13-14. A sphere of radius R has a charge density of $\rho = \alpha r$ where α is a constant.

13. The total charge on the sphere is:

A. $q = \frac{2\alpha\pi R^3}{3}$

B. $q = \frac{2\alpha\pi r^3}{3}$

C. $q = \alpha\pi R^4$

D. $q = \alpha\pi r^4$

E. None of the above.

14. Use Gauss's Law to find an equation for the magnitude of the electric field outside the sphere. Mark the correct answer.

A. $E = \frac{\alpha R^4}{4\varepsilon_0 r^2}$

B. $E = \frac{\alpha r^2}{4\varepsilon_0}$

C. $E = \frac{\alpha R^3}{3\varepsilon_0 r}$

D. $E = \frac{\alpha r^2}{3\varepsilon_0}$

E. None of the above.

15-16. A plane of charge has a surface charge density $\sigma = 4.00\ \mu C / m^2$.

☐ 15. If the surface area of one side of a Gaussian surface is *A*, the flux through the Gaussian surface is:

A. *E/A*
B. *E/2A*
C. *2E/A*
D. *EA*
E. *2EA*
F. *EA/2*
G. None of the above

☐ 16. The magnitude of the electric field is what? Mark the first significant digit.

17-18. A long, cylindrical wire of radius *R* has a current density of $j = \alpha r^3$ where α is a constant. Use Ampère's Law to find the magnitude of the magnetic field at a radius *r* inside the wire.

☐ 17. Which of the following is the best expression for the line integral?

A. $\Lambda = 2\pi r$

B. $\Lambda = 2\pi r B$

C. $\Lambda = 2\pi R$

D. $\Lambda = 2\pi R B$

E. $\Lambda = \pi r^2$

F. $\Lambda = \pi r^2 B$

G. $\Lambda = \pi R^2$

H. $\Lambda = \pi R^2 B$

I. None of the above.

☐ 18. Use Ampère's Law to find the magnitude of the magnetic field at a radius *r* inside the wire.

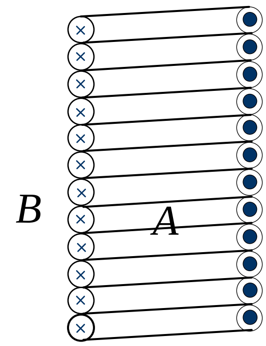
A. $B = \mu_0 \dfrac{\alpha r^4}{6}$ B. $B = \mu_0 \dfrac{\alpha R^6}{6r^2}$

C. $B = \mu_0 \dfrac{\alpha r^4}{5}$ D. $B = \mu_0 \dfrac{\alpha R^5}{5r}$

E. None of the above.

19-20. Use Ampère's Law to find the magnitude of the magnetic field as a function of *r* inside a solenoid.

19. A solenoid is shown in the drawing to the right. Current passes out of the screen on the right-hand side. The directions of the magnetic field inside and outside the solenoid (points *A* and *B*) are:

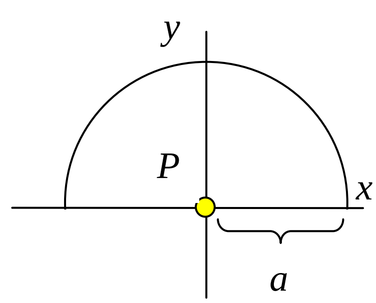


A. no field, no field
B. no field, down
C. down, no field
D. down, down
E. down, up

20. If the solenoid is made of 200 turns of wire, the length of the solenoid is 8.00 cm, and a current of 0.24 A passes through the coil, what is the magnitude of the magnetic field at *A*.

Mark the first significant digit of your answer.

21-22. A semicircular segment of wire of radius *a* has its center at the origin of a coordinate system as shown to the right. A current passes counterclockwise through the loop. Use the Biot-Savart Law to find the magnetic field at the origin.



The Biot-Savart Law states: $d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{\ell}\times\vec{R}}{R^3}$.

21. An appropriate expression for $\vec{R}$ would be:

A. $\vec{R} = a(\cos\theta'\,\hat{x} + \sin\theta'\,\hat{y})$ B. $\vec{R} = a(\cos\theta'\,\hat{x} - \sin\theta'\,\hat{y})$
C. $\vec{R} = a(-\cos\theta'\,\hat{x} + \sin\theta'\,\hat{y})$ D. $\vec{R} = a(-\cos\theta'\,\hat{x} - \sin\theta'\,\hat{y})$
E. $\vec{R} = a(\sin\theta'\,\hat{x} + \cos\theta'\,\hat{y})$ F. $\vec{R} = a(\sin\theta'\,\hat{x} - \cos\theta'\,\hat{y})$
G. $\vec{R} = a(-\sin\theta'\,\hat{x} + \cos\theta'\,\hat{y})$ H. $\vec{R} = a(-\sin\theta'\,\hat{x} - \cos\theta'\,\hat{y})$

22. An appropriate expression for $d\vec{\ell}$ would be:

A. $d\vec{\ell} = ad\theta'(\cos\theta'\,\hat{x} + \sin\theta'\,\hat{y})$ B. $d\vec{\ell} = ad\theta'(\cos\theta'\,\hat{x} - \sin\theta'\,\hat{y})$
C. $d\vec{\ell} = ad\theta'(-\cos\theta'\,\hat{x} + \sin\theta'\,\hat{y})$ D. $d\vec{\ell} = ad\theta'(-\cos\theta'\,\hat{x} - \sin\theta'\,\hat{y})$
E. $d\vec{\ell} = ad\theta'(\sin\theta'\,\hat{x} + \cos\theta'\,\hat{y})$ F. $d\vec{\ell} = ad\theta'(\sin\theta'\,\hat{x} - \cos\theta'\,\hat{y})$
G. $d\vec{\ell} = ad\theta'(-\sin\theta'\,\hat{x} + \cos\theta'\,\hat{y})$ H. $d\vec{\ell} = ad\theta'(-\sin\theta'\,\hat{x} - \cos\theta'\,\hat{y})$

23-24. A magnetic field is given by the expression $\vec{B} = \alpha x^2 y\hat{x} + \beta xy^2\hat{y}$ where α is a constant.

23. What is true of α and β?

A. $\alpha = 0$ B. $\beta = 0$

C. $\beta = \alpha$ D. $\beta = -\alpha$

E. $\beta = \frac{1}{\alpha}$ F. $\beta = -\frac{1}{\alpha}$

24. Find an expression for the current density.

A. $\mu_0 \vec{j} = (2\beta xy - 2\alpha xy)\hat{z}$ B. $\mu_0 \vec{j} = -(2\beta xy - 2\alpha xy)\hat{z}$

C. $\vec{j} = 2\beta xy\hat{x} + 2\alpha xy\hat{y}$ D. $\vec{j} = -2\beta xy\hat{x} - 2\alpha xy\hat{y}$

E. $\mu_0 \vec{j} = (\beta y^2 - \alpha x^2)\hat{z}$ F. $\mu_0 \vec{j} = -(\beta y^2 - \alpha x^2)\hat{z}$

25-26. Consider Ampère's Law and Gauss's Law of Electricity.

25. Which of the following is not correct?

A. The net number of perpendicular surfaces pierced by an Amperian loop is proportional to the current passing through the loop.

B. The source of looping magnetic field lines is electric current.

C. The curl of the magnetic field at a point is in the direction of the current passing through that point.

D. The magnetic line integral around an Amperian loop is proportional to the current passing through the loop.

E. $\nabla \times \vec{B} = \mu_0 i$

F. $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc}$

G. $\Lambda_B = \mu_0 \int \vec{j} \cdot d\vec{A}$

26. Which of the following is not correct?

A. The net number of field lines passing through a Gaussian surface is proportional to the charge enclosed by the surface.

B. The source of spreading electric field lines is electric charge.

C. The divergence of the electric field at a point is in the direction the charge increases most rapidly.

D. The electric flux through a Gaussian surface is proportional to the charge contained in the surface.

E. $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

F. $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

G. $\varepsilon_0 \Phi_E = \int \rho \, dV$

Selected Answers:

1.D 2.E 3.A 4.D 5.A 6.D 7. A 8.C 9.B 10.C

11-12. $\tau = RC = 48\mu s, \quad i(t) = \frac{V_0}{R} e^{-t/(RC)}, i = \frac{V_0}{R} e^{-1} = 61mA$

13-14.
$$\int_0^R \alpha r 4\pi r^2 dr = \frac{4\pi\alpha R^4}{4} = \pi\alpha R^4$$
$$4\pi r^2 E = \frac{1}{\varepsilon_0}\int_0^R \alpha r 4\pi r^2 dr, \quad E = \frac{\alpha R^4}{4\varepsilon_0 r^2}$$

15-16. $\Phi = 2EA = \frac{\sigma A}{\varepsilon_0}, \quad E = \frac{\sigma}{2\varepsilon_0} = 2.26\times 10^5\ V/m$

17-18. $\Lambda = 2\pi r B = \mu_0 \int_0^r \alpha r^3 2\pi r dr, \quad B = \frac{\mu_0 \alpha r^4}{5}$

19-20. $BL = \mu_0 N i, \quad B = \frac{\mu_0 N i}{L} = 7.54\times 10^{-4}\ T$

21-22. $\vec{r} = 0, \vec{r}\,' = a\cos\theta'\,\hat{x} + a\sin\theta'\,\hat{y}, \vec{R} = -\vec{r}\,', \quad d\vec{\ell} = -ad\theta'\sin\theta'\,\hat{x} + ad\theta'\cos\theta'\,\hat{y}$

23-24.

$B_x = \alpha x^2 y \quad B_y = \beta x y^2$

$\nabla\cdot\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_x}{\partial x} = 0 \Rightarrow 2\alpha x y + 2\beta y x = 0 \Rightarrow \beta = -\alpha$

$\nabla\times\vec{B} = \mu_0 \vec{j} = \hat{z}\left(\beta y^2 - \alpha x^2\right)$

25-26. E, C