Physics 123 Unit #5 Review

I. Definitions & Facts

world line	propagator	rest energy
total energy	kinetic energy	β and γ
inertial reference frame	Lorentz boost	relativistic invariants
half-life	scattering angle	Galilean relativity
Correspondence Principle		

II. Mathematics & Tools

Be able to do matrix multiplication. Understand how matrices function as operators on vectors. Be able to write space-time and energy-momentum four-dimensional vectors.

Know how to take inner (dot) products of four-vectors. Inner products are relativistic invariants. Know what the metric tensor is (in flat space-time).

III. Basic Concepts

Experimentally, energy is not just $p = m\,v$.

Propagators are tangents to the world line that provide a mechanism for determining the world line. The energy-momentum four-vector is the relativistic propagator .

By assuming that the time component of force must change in the same way as the classical kinetic energy, we can derive the correct relationship for relativistic momentum.

Moving objects are heavy and contracted in the direction of motion. Moving clocks run slow. A factor of γ relates each rest and moving measurement.

Be able to construct Lorentz boost matrices and their inverses for motion along any axis.

The two postulates of relativity are 1) Galilean relativity holds, 2) the speed of light in vacuum is a constant in any reference frame.

IV. Equations to Memorize

Rest energy: $E_0 = mc^2$

Total energy: $E = m\gamma c^2$

β and γ: $\beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

Kinetic energy: $K = E - E_0$

Momentum magnitude: $pc = mc^2\beta\gamma$

Space-time vector (Four-position): $\boldsymbol{x} = \begin{pmatrix} w=ct \\ x \\ y \\ z \end{pmatrix}$

Energy-momentum four-vector (Four-momentum): $\boldsymbol{E} = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$

Energy-momentum relationship: $E^2 = p^2c^2 + E_0^2$

Physics 123
Unit 5 Reading and Homework Assignments

Introduction: Particularly note Einstein's two postulates of relativity.

1–1: This should be a review of how we can represent systems of linear equations with matrices. If you are rusty with matrices, then you should carefully review this material, as we will use matrices extensively. On the other hand, we won't be doing a lot of complicated things with matrices, so you should find the material fairly easy.

1–2: I put all the gory detail in, in case anyone feels a little lost as to what's going on. The bottom line is the definition of matrix multiplication. We'll go over this process in class in more detail, so don't panic if it doesn't seem familiar at first reading.

1–3: We are going to use matrices as "operators." That is, matrices will become devices which change states, such as the space-time coordinates or energy and momentum, of objects. Look over the examples carefully.

2–1. Momentum and its conservation are concepts we take for granted. We need to break ourselves from the conclusions that came from 17th century experiments.

2–2. This is a very important idea that guides all new theories.

2–3. There is no new physics here, only old physics presented from a new viewpoint. Spend time in this section to be sure you understand it.

2–4. We wish to create a "propagator," a vector that is a tangent to the world line in space-time. If we can find the propagator at any point in space, we can propagate the world line one tiny step at a time. Refer to Appendix A for a concrete example of how we can use the propagator to generate the world line.

2–5. This section basically defines the energy-momentum vector. We will use this vector a lot.

2–6. The relativistic energy-momentum relationship is really important, but be sure that you understand how we get there. The subtle difference between 0 and ΔK for the change in the time component of the energy-momentum vector is all that separates Newtonian mechanics from relativistic mechanics.

2–7. Now we show that the experimental result for relativistic momentum is a consequence of our choice of how the time component of the energy-momentum vector changes.

2–8. This is a recap of the relativistic energies and momentum. Know these relationships well.

2–9. A little philosophy...

3–1. Know the ideas here well.

3–2. This is a bit of mathematical detail that you don't need to reproduce, but follow the logic through carefully. If you don't the Lorentz transformation will remain a mystery. The crucial results are equations (4–1) and (4–2).

3–3. The same transformation applies to space-time as to energy-momentum. Since they are intimately related, this isn't surprising.

3–4 and 3–5. These are two applications of Lorentz transformations. We shall see more later.

4–1. We show that the speed of light is a constant with respect to any observer. Note also the way we set up problems in terms of events. If you follow this pattern, you'll run less danger of confusion.

4–2. Invariants are often very important, as they tend be fundamental quantities. We won't develop further the ideas of this section, however.

4–3. These are very important examples. Be able to reproduce each one.

Appendix A. Be sure you understand the process here. It isn't crucial to the development that follow, but you'll feel ill at ease through the entire section if you don't spend the time you need to grasp these ideas.

B–1. Appendix B is optional, but you will probably find it to be interesting. This chapter is to give you a feeling of how relativity relates to forces. Notice how we can construct a four-force that obeys Lorentz transformations. The results are interesting, and the manipulations of four-vectors are useful, but you might find the material a bit abstract.

B–2. If you have not had electricity and magnetism, some of these ideas may seem to be pulled out of the air. In the end, we find the magnetic field of a current-carrying wire is just the result of relativity. The magnetic field is not fundamental at all, nor are the equations that relate electric and magnetic fields, Maxwell's Equations. These are natural consequences of the fundamental relationships between space and time embodied in relativity.

B–3. If magnetic fields can be derived from electrostatics and relativity, then gravity should show magnetic effects as well. This is one more example of how we can manipulate four-vectors with Lorentz transformations.

Physics 123 – Section 2
Sample Exam #5

● Possibly useful information:

$c = 3.00 \times 10^8$ m / s
For a photon, $E = hf = \hbar\omega = pc$

Terminology

1. (5 points) rest energy

2. (5 points) inertial reference frame

3. (5 points) relativistic invariant

4. (5 points) Correspondence Principle (as applied to relativity)

Conceptual Applications

5. (5 points) Which of the following was **not** used as a postulate by either by Einstein or Rees in deriving special relativity?

A. Mass can be converted into energy.

B. Galilean relativity is valid.

C. The speed of light is a constant.

D. Space and time should enter into equations in a similar fashion.

E. All of the above are postulates of relativity.

6. (5 points) Which of the following **is** a consequence of special relativity:

A. inertial mass equals gravitational mass

B. light traveling away from the earth undergoes a gravitational red shift

C. time must be replaced by ct to have dimensions of length

D. the magnetic field

E. none of the above are consequences of special relativity

Equations and Tools

7. Multiply the following matrices:

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} =$$

For each of the following equations:

A) Tell what each symbol means

B) Write a short problem which can be solved with this equation

The problem must be a word problem, not just "*a=5, b=4, what is c?*"

8. (5 points) $E_0 = mc^2$

Problems

We can usually evaluate how well you understand the physics of a problem by the way you attempt to solve the problem. Occasionally, however, you may be unable to work a problem or a part of a problem even if you do understand the physics. If you cannot work a problem, please do the following so that you may receive partial credit:

1. Discuss the basic physics concepts involved. Include enough detail that the grader will know if you understand the physics.
2. Write down the equation(s) you need to solve the problem. Explain the meaning of each symbol in the equations.
3. Explain as much of the problem as you understand.
4. Explain why you feel you have difficulties with the problem.

9. (25 points) (a) Discuss propagators. Describe what they are, how they function to generate world lines, what causes propagators to change from point to point in space, and the mathematical forms of the non-relativistic and relativistic propagators. (18 points)

(b) Why are there significant differences between Newtonian physics and relativistic physics even though the propagators are so similar? (7 points)

10. (50 points) Compton Scattering is one of the important interactions of gamma-rays (high-energy photons) with matter. Compton Scattering can be treated as the elastic scattering of a photon with a stationary electron. In this problem you will derive the formula that gave Arthur Compton the 1927 Nobel Prize in Physics.

(a) Assume that the energy of the incident gamma-ray is E_γ. Let the rest energy of the electron be written as E_0. Write down energy-momentum vectors for both particles prior to collision. (7 points)

(b) Find the Lorentz transformation which takes four-vectors from the laboratory frame where the electron is at rest, to the zero-momentum frame of the photon-electron system. Find β of the boost in terms of E_γ. and E_0. (10 points)

(c) Using invariants, show that *W*, the total energy in the zero-momentum frame is $W = \sqrt{E^2 - E_\gamma^2}$. Here $E = E_\gamma + E_0$.is the total energy in the lab frame. (8 points)

(d) Now show that

$$\gamma = \frac{E}{W} \text{ and } \beta\gamma = \frac{E_\gamma}{W}.$$

(6 points)

(e) Find the energy-momentum vector of the electron before the collision in the zero-momentum frame. (6 points)

(f) The collision can do nothing to the electron in the zero-momentum frame except change the direction of its motion. For the special case where the electron has the maximum possible kinetic energy in the laboratory, write the energy-momentum four-vector for the electron **after** the collision in the zero-momentum frame. (5 points)

(g) Show that in the laboratory, the electron's maximum kinetic energy is $K_{max} = 2E_0 \frac{E^2_\gamma}{W^2}$. (8 points)

Answers

Terminology

1. *The energy equivalent of a particle's mass.*
2. *A set of non-accelerating coordinate axes and clocks used to measure the space-time coordinates of an event.*
3. *A quantity which is the same in any inertial reference frame.*
4. *Any relativistic result must reduce to the corresponding non-relativistic result in the limit of small velocity.*

Conceptual Applications

5. A. 6. D.

Equations and Tools

7.
$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 3 & 4 & 3 \\ 6 & 0 & 6 \end{pmatrix}$$

8. E_0 *is total energy*
m *is the mass*
c *is the speed of light*

How much energy would be produced if you converted all the mass of a 10.0 g rock into energy?

9. (a) *A minimum response should include the following:*

- *A propagator is a four-dimensional vector that is tangent to (points in the direction of) the world line at any point in space.*
- *Given one point on a world line, a nearby point is obtained by moving a small distance along the direction of the propagator.*
- *Forces change the propagator from point to point.*
- *The non-relativistic propagator can be written in one of the following forms:* $\boldsymbol{P} = \begin{pmatrix} E_0 \\ p_x c \\ p_y c \\ p_z x \end{pmatrix} = \begin{pmatrix} mc^2 \\ mv_x c \\ mv_y c \\ mv_z c \end{pmatrix}$

(Any quantity proportional to these is acceptable.)

(b) *The time component of the non-relativistic propagator is fixed. It is not influenced at all by spatial variables. However, relativistically, the time component of the propagator is proportional to γ, so it has a dependence on the momenta. This causes time to be interrelated with space relativistically, rather than being a completely separate entity.*

10. (a)
$$\boldsymbol{E}_\gamma = \begin{pmatrix} E_\gamma \\ 0 \\ 0 \\ E_\gamma \end{pmatrix}, \qquad \boldsymbol{E}_e = \begin{pmatrix} E_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

It is acceptable to have the momentum of the photon in a different direction.

(b)
$$\boldsymbol{L} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$\begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} E_\gamma + E_0 \\ 0 \\ 0 \\ E_\gamma \end{pmatrix} = \begin{pmatrix} \gamma(E_\gamma + E_0) - \beta\gamma E_\gamma \\ 0 \\ 0 \\ -\beta\gamma(E_\gamma + E_0) + \gamma E_\gamma \end{pmatrix} = \begin{pmatrix} W \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\beta = \frac{E_\gamma}{E_\gamma + E_0}$$

(c)

$$\boldsymbol{E}_{total} = \begin{pmatrix} E \\ 0 \\ 0 \\ E_\gamma \end{pmatrix}, \qquad \boldsymbol{E}'_{total} = \begin{pmatrix} W \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In **S** , $\boldsymbol{E}_{total} \cdot \boldsymbol{E}_{total} = E^2 - E^2_\gamma$ is an invarainat

In **S** $'$, $\boldsymbol{E}'_{total} \cdot \boldsymbol{E}'_{total} = W^2$

$$W = \sqrt{E^2 - E^2_\gamma}$$

(d)

$$\beta = \frac{E_\gamma}{E_0 + E_\gamma} = \frac{E_\gamma}{E}, \quad 1 - \beta^2 = \frac{E^2 - E_\gamma^2}{E^2} = \frac{W^2}{E^2}, \quad \gamma^2 = \frac{1}{1-\beta^2} = \frac{E^2}{W^2} \quad \gamma = \frac{E}{W}, \quad \beta\gamma = \frac{E_\gamma}{W}.$$

(e)

$$\boldsymbol{E}'_e = \boldsymbol{L}\boldsymbol{E}_e =$$

$$\frac{1}{W}\begin{pmatrix} E & 0 & 0 & -E_\gamma \\ 0 & W & 0 & 0 \\ 0 & 0 & W & 0 \\ -E_\gamma & 0 & 0 & E \end{pmatrix}\begin{pmatrix} E_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{E_0}{W}\begin{pmatrix} E \\ 0 \\ 0 \\ -E_\gamma \end{pmatrix}$$

(f) *This will be when the electron moves in the +*z *direction. Hence, using a ~ to denote quantities after the collision:*

$$\tilde{\boldsymbol{E}}'_e = \frac{E_0}{W}\begin{pmatrix} E \\ 0 \\ 0 \\ +E_\gamma \end{pmatrix}$$

(g) *We do an inverse transformation back to the lab:*

$$\tilde{\boldsymbol{E}}_e = \boldsymbol{L}^{-1}\tilde{\boldsymbol{E}}'_e =$$

$$\frac{1}{W}\begin{pmatrix} E & 0 & 0 & +E_\gamma \\ 0 & W & 0 & 0 \\ 0 & 0 & W & 0 \\ +E_\gamma & 0 & 0 & E \end{pmatrix}\frac{E_0}{W}\begin{pmatrix} E \\ 0 \\ 0 \\ E_\gamma \end{pmatrix} = \frac{E_0}{W^2}\begin{pmatrix} E^2 + E^2_\gamma \\ 0 \\ 0 \\ 2E_0 E_\gamma \end{pmatrix}$$

$$K_{\max} = E - E_0 = \frac{E_0}{W^2}(E^2 + E^2_\gamma) - E_0$$

$$K_{\max} = \frac{E_0}{W^2}(E^2 + E^2_\gamma - W^2)$$

$$K_{\max} = 2E_0 \frac{E^2_\gamma}{W^2}$$

Physics 123 Unit Summary
Unit #5

ID Number: ____________ Unit Score: (sum all boxes below): [ ]

1. Homework problems completed on time ________ × 10 = _________

   Homework problems completed on time ________ × 8 = _________

   Sum of previous lines ________ ÷ 30 problems = __________, average score per problem

   Homework score: Average score per problem × 2.5 = [ ] (Maximum = 25)

2. Hours

Item	Hours	Item	Hours
____________________	________	____________________	________
____________________	________	____________________	________
____________________	________	____________________	________
____________________	________	____________________	________

Total Hours (expected = 21, maximum = 36): ________

Standard Work Score: Total Standard Hours × 1.94 = [ ] (Maximum = 70)

4. Reading Checks

Reading score: $10 \times \frac{\#\ \text{correct}}{8}$ = [ ] (Maximum = 10)

4. Quizzes

Quiz score: $10 \times \frac{\#\ \text{correct}}{8}$ = [ ] (Maximum = 10)

5. Walk-in Labs

Lab score: = [15] (Maximum = 15)