

PHYSICS 221

PROBABILITY EXERCISES

Two rules:

- A. The probability that both of two **unrelated** events occur is the product of the two individual probabilities.
- B. The probability that either of two **mutually exclusive** events occurs is the sum of the two individual probabilities.

1. What is the probability of throwing exactly one head when tossing three coins? What is the probability of throwing at least one head?

- With a simple system like this, it is easy to list all the possible outcomes and count the desired outcomes. The possible results are HHH, HHT, HTH, THH, TTH, THT, HTT, TTT.

Answers: 3/8, 7/8.

2. Using the rules, what is the probability of throwing three heads? Of all the coins being the same?

- Answers: 1/8, 1/4.

3. The probability of throwing at least one head is 7/8. The probability of throwing exactly two heads is 3/8. What is the probability of throwing either a) at least one head or b) two heads?

- We're tempted to say the probability is the sum of a) and b). This gives $10/8 > 1!$ The difficulty is that we're double counting. When we throw two heads, we're also throwing at least one head. Be careful of cases which are more subtle.

Answer: 7/8.

4. What is the probability of throwing exactly one head when tossing ten coins? What is the probability of throwing at least one head?

- Since there are 1024 possibilities, it's hard to write all the possibilities, so we need to be clever. There are 10 ways to throw exactly one head. There is only one way to throw **no** heads.

Answers: 10/1024, 1023/1024.

5. What is the probability of throwing exactly two heads when tossing ten coins?

- The probability of the first coin being a head is $1/2$. The probability of exactly one of the next nine being a head is $9/512$. The probability that the first coin is a tail is $1/2$, that the next is a head is $1/2$, and that exactly one of the next eight is a head is $8/256$, etc. $1/2 \times 9/512 + 1/2 \times 1/2 \times 8/256 + \dots$. If you remember your math, a quicker way to determine the number of ways we can have two heads in ten throws is to take the combination of 10 things taken 2 at a time:

$$C\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10 \times 9}{2} = 45$$

Answer: 45/1024.

6. What is the probability that the sum of two dice is 3?

- Here, the easiest way is to count.

Answer: 1/18.

7. What is the probability of rolling at least one 6 when tossing eight dice?

- What's wrong with $8 \times 1/6$? When you see "at least" think "not none." That is, the probability of tossing **no** sixes in one throw is $5/6$. In eight throws it is $(5/6)^8$.

Answer: 0.7674 (or 76.74%).

8. What is the probability of dealing five cards (from a full deck) all of the same suit? What is the probability that all five cards are hearts?

- The methods are similar, except that the total number of cards decreases after every draw. The probability of the first card being the same suit as itself is 1. (That is, any card is OK to start.) The probability of the second being the same suit as the first is $12/51$, etc.

Answer: 0.00198, 0.000495

9. What is the probability of dealing four cards (from a full deck) all of the different suits?

- Answer: 0.01055 (**not** 0.01056 ...)

10. In the game of blackjack 10, J, Q, and K count as 10 points and A as 11 points (I think). What is the probability of getting 21 points in two cards if they're drawn from a full deck? If the deck has already had the following cards drawn: $2\heartsuit 5\clubsuit 7\heartsuit 8\diamond 10\heartsuit K\spadesuit K\clubsuit A\diamond$?

- Remember that the 10 point card can be either the first or second card drawn.

Answer: 0.0965, 0.0412