

Physics 123 Relativity Review

I. Definitions & Facts

reference frame	rest energy	correspondence principle
inertial reference frame	kinetic energy	β and γ
the principle of relativity	energy or total energy	effective mass

II. Mathematics and Tools

Know the ranges of β and γ .

III. Basic concepts

We can formulate relativity in a four-dimensional space with time as a fourth variable.

When we write a space-time four vector, we use ct instead of just t for the time component so that this component will have dimensions of length. Without relativity, c is an arbitrary constant.

We see that momentum “moves” an object from one position to another and energy ($E = mc^2$) “moves” an object from one time to another.

Relativity comes in by 1) allowing momentum to be $p = m\gamma v$. This causes the energy ($E = m\gamma c^2$) to change when velocity changes, so ΔE is no longer zero.

As pc increases, it can never get larger than E . If $pc = E$, $m\gamma v c = m\gamma c^2$, so $v = c$. The constant c must be the fastest speed any object can travel, the speed of light.

An object moving at high speed relative to an observer has 1) its length in the direction of motion contract by a factor of γ , 2) its clocks tick slowly by a factor of γ , and 3) its effective mass ($m\gamma$) increase by a factor of γ .

Lorentz transformations relate how any four-vector transforms between observers in two different inertial reference frames. Know how to use these equations, but you do not need to memorize them. If S' moves at speed β in the $+x$ direction with respect to S ,

$$\begin{aligned} ct' &= \gamma ct - \beta \gamma x & ct &= \gamma ct' + \beta \gamma x' \\ x' &= \gamma x - \beta \gamma ct & x &= \gamma x' + \beta \gamma ct' \\ y' &= y & y &= y' \\ z' &= z & z &= z' \end{aligned}$$

Two important four-vectors you should know are:

space-time four-vector (ct, x, y, z)

energy-momentum four-vector $(E, p_x c, p_y c, p_z c)$

Einstein based the original derivation of special relativity on two postulates:

- 1) The principle of relativity: the laws of physics are the same in all inertial reference frames
- 2) Constant speed of light: the speed of light in vacuum is measured to be the same by any observer, independent of the motion of the source or observer

General relativity is a theory of gravity based on the notion that mass changes the curvature of space-time, and the curvature of space-time, in turn affects the motion of matter.

The principle of equivalence suggests that there is no difference between measurements made by an observer in an accelerating reference frame and measurements made by an observer in an equivalent gravitational field.

IV. Equations to memorize

$$\text{Beta and gamma: } \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\text{Rest energy: } E_0 = mc^2$$

$$\text{Total energy: } E = mc^2 \gamma$$

$$\text{Momentum: } pc = mc^2 \beta \gamma$$

$$\text{Energy-momentum relationship: } E^2 = (pc)^2 + E_0^2$$

$$\text{Effective mass: } m_{\text{eff}} = m \gamma$$

$$\text{Kinetic energy: } K = E - E_0$$

III. Sample Problems

Not yet available