Physics 123 Quantum Mechanics Review

I. Definitions & Facts

blackbody radiation
de Broglie wavelength
work function
quantization
Compton scattering
photon
photoelectric effect
uncertainty principle
cutoff frequency
principle of complimentarity
wave function
probability density
normalization
operators
momentum operator
expectation value
emission spectrum
absorption spectrum
ionization energy
quantum number
n, l, m_l, m_s
shell
orbital
Bohr model
spin
ionic bonds
covalent bonds
van der Waals bonds
hydrogen bonds
band gaps
amu
half life
decay constant
hadrons
baryons
mesons
leptons
quarks
flavors: udscbt
colors: rgb

II. Basic concepts

Planck first proposed the quantization of oscillators (atoms) to explain blackbody radiation

Einstein suggest that always matter emits and absorbs light in bundles called photons with energy $E = hf$.

When an electron is emitted from a metal surface, the photon's energy is completely absorbed. An energy of at least the work function φ is required to remove the electron from the surface. Additional energy remains as kinetic energy of the ejected electron.

In the photoelectric effect, if the energy of the photon is less than the work function (or the frequency is less than the cutoff frequency), no electrons can be ejected from the surface.

X-rays are high energy photons. The wavelength of x-rays changes when they are scattered from electrons in matter. This is called Compton scattering. The change in wavelength can be calculated by considering conservation of momentum and energy in a photon-electron collision.

Evan as light has particle-like attributes, matter has wave-like attributes. The wavelength of a particle is given by the de Broglie formula.

Because of the wave nature of matter, some pairs of physical quantities can inherently be measured simultaneously only within certain degrees of accuracy. For example, If we measure the position of a particle with high precision, we can measure the corresponding component of its momentum only with a large uncertainty. This is called the Heisenberg uncertainty principle.

The wave function is a complex function that describes the probability of a system being in a state. We will deal mostly with spatial wave functions in one dimension where the probability of finding a particle in a region Δx is given by $P(x)\Delta x = \psi^*(x)\psi(x)\Delta x = |\psi(x)|^2 \ \Delta x$.

In one dimension, wave functions are normalized so that $\int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx = 1$. The probability of finding a particle in the region $x=a$ to b is then: $P(a \to b) = \int_a^b \psi^*(x)\psi(x)dx$

Physical quantities such as energy and momentum can be described by operators. If a state has a definite value of some such quantity, the operator satisfies the equation $\mathbf{O}\psi = O\psi$ where **O** is the operator and O is the measured value of the physical quantity.

A photon of light is emitted by an atom when it makes a transition from one energy level to another. The energy of the photon is the difference in energy between the atomic energy levels.

Bohr atom: planetary orbits, angular momentum (incorrectly quantized), lead to the correct energy levels.

The Schrödinger equation has for solutions certain wave functions and their corresponding energies. Since only certain specific energies are allowed, energy is quantized.

In a one-dimensional infinite well from x=0 to L, the solutions of Schrödinger's equation are like standing waves on a string. The wave functions are $\psi(x) = A\sin(kx)$ where $k = n\pi / L$. The allowed energies are $E_n = \hbar^2 k^2 /(2m)$.

The Bohr model was based on quantized angular momentum or equivalently, standing waves in circular orbits. The correct energy levels were given, but electrons do not have planetary orbits and Bohr's quantization condition was incorrect.

Schrödinger's equation leads to solutions that depend on certain integers called quantum numbers. These are:

- Principle quantum number: n, distinguishes electron shells and energies. n = 1,2,3… corresponds to K, L, M shells of the hydrogen atom
- Angular momentum quantum number: l = 0, 1 ,…, n − 1. The total angular momentum is $L = \sqrt{l(l+1)}\hbar$. l = 0, 1, 2, 3, 4, … corresponds to s, p, d, f, g subshells.
- Orbital magnetic quantum number; $m_l = -l, -l+1, \ldots, l$. The projection of the angular momentum along the z axis is $L = m_l \hbar$.
- Spin magnetic quantum number: $m_s = \pm½$. Spin up or spin down. Total spin for an electron or proton is ½.

Orbitals represent the probability distribution of electrons in an atom. They are obtained by taking $|\psi|^2$. Orbitals are labeled by n and the letter corresponding to l, such as 1s, 2p, etc.

Bonds: ionic, electron exchanged between ions; covalent, atoms share electrons between nuclei that attract both nuclei; van der Waals, some regions of molecules are more positive and some more negative, so there is weak attraction; hydrogen, similar to van der Waals except the positive regions are "bare" protons that bond more strongly.

In solids, energy levels turn into bands of many closely spaced energy levels. If there are empty levels available to electrons with little excitation, the material is a conductor. If there is a large band gap between a filled band and an unfilled band, the material is an insulator.

For lighter elements stable nuclei have $N = Z$. For heavier elements the Coulomb repulsion makes proton levels spread out and the nucleus has $N > Z$.

Binding energy per nucleon is largest in the vicinity of iron.

The liquid drop model is used to find an approximate expression for nuclear binding energies.

Be able to balance α, β, and γ decay equations. (Find A, Z, N.)

Dirac thought of antiparticles as particles elevated from a negative-energy sea into a positive-energy band.

The four fundamental forces in order of strength are strong, electromagnetic, weak, and gravitational.

Be able to estimate the range of a force if you are given the mass of the particle that mediates the force. The mediating particles for the forces are: strong – mesons (gluons), electgromagnetic – photons, weak – W and Z, gravity – gravitons.

Electromagnetic and weak forces can be accurately described by summing over all possible Feynmann diagrams. Feynmann diagrams are interpreted in terms of the exchange of virtual particles.

Charge, baryon number, lepton number, muon number, and electron number are always conserved.

Strangeness is conserved in strong interactions (which often produce strange particles) but one unit of strangeness can be gained or lost in weak interactions, by which strange particles usually decay.

You need to know the baryon number, muon number, electron number, and lepton number of protons, neutrons, pions. muons, electrons, and all types of neutrinos. Know that charge, strangeness, baryon number, etc. have opposite signs for particles and antiparticles.

Baryons are composed of three quarks, one red, one green, and one blue. Antibaryons are composed of three antiquarks.

Confinement: the forces between particles of different color gets larger with distance. This means that particles form in colorless combinations and that we can never expect to see free quarks.

Camouflage: gluons, the particles that mediate the color force, carry color themselves. That means that the core quark is constantly changing colors. Since like colors repel and unlike colors attract, nearby quarks experience no net force, as the colors are indeterminate.

Mesons are composed of one quark and one antiquark. The colors must be $b\bar{b}$, etc.

There are many unanswered questions such as why particles have the masses they do. The key to understanding mass may be the Higgs boson, called the “God particle” in the popular press.

String theory suggests that particles can be considered to be vibrations on strings that have a length of order 10^{-35} m. The present string theory views strings as closed, spinning, and living in 10 dimensions. The theory also suggests the existence of very heavy “supersymmetric” partners to known particles. It may be that 11-dimensional membranes may be needed instead of 10-dimensional strings. The theory does seem to unify gravity with the strong and electroweak forces.

III. Equations to know

Planck’s quantization condition: $E = nhf$

Photon energy and momentum: $E = hf \quad p = \frac{h}{\lambda}$

Photoelectric effect: $hf = K_{\max} + \phi$

Cutoff frequency: $hf_{cutoff} = \phi$

de Broglie’s equation: $p = \frac{h}{\lambda}$

Space-momentum uncertainty relation: $\Delta x \Delta p_x \geq \frac{\hbar}{2}$

Time-energy uncertainty relation: $\Delta t \Delta E \geq \frac{\hbar}{2}$

Momentum operator in the *x* direction: $\mathbf{p}_x - i\hbar\frac{\partial}{\partial x}$

Hamiltonian or total energy operator in one dimension: $\mathbf{H} = \mathbf{K} + \mathbf{U} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$

Schrödinger equation: $\mathbf{H}\psi(x) = E\psi(x)$

Bohr radii (most probable radii) for H: $r = a_0 r^2$ (You don't need to memorize a_0.)

Hydrogen energy levels: $E_n = -\frac{E_0}{r^2}$ (You don't need to memorize E_0.)

Nuclear radius: $r = r_0 A^{1/3}$, r_0 =1.2 fm

Nuclear binding energy: ((mass of Z*H + N*n) – (mass of atom))c^2

Q value of decay or reaction: (mass of original particles – mass of final particles)c^2

Nuclear decay: $\frac{dN}{dt} = -\lambda t = -R \qquad N = N_0 e^{-\lambda t} \quad T_{1/2}\lambda = \ln 2$

IV. Problems

Review the quiz questions and homework questions.