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Challenge Problems 5-7

“Elliptical” Heat Engine

This problem has a number of subparts and will count as three separate challenge problems. You may work one, two, or all three problems, as you choose. It requires the use of a spreadsheet or a programming language that does the same thing. A very basic understanding of spreadsheets is all that is necessary, but feel free to contact Dr. Rees if you need some help with spreadsheet.

A reversible heat engine operates in a cycle that is described by the equation:

$$\frac{(P-P_0)^2}{P_R^2}+\frac{(V-V_0)^2}{V_R^2}=1$$

where P_0 and P_R are constants with units of pressure and V_0 and V_R are constants with units of volume. Note that the path on a PV diagram is in the shape of an ellipse. You will find the efficiency of this cycle in a series of steps that we will break into three individual problems.

Problem 5.

First, we will simplify the calculation by making a change of variables. Let us define

$$y=\frac{P}{P_R},\quad y_0=\frac{P_0}{P_R},\quad x=\frac{V}{V_R},\quad x_0=\frac{V_0}{V_R}.$$

Clearly, this reduces the equation to

$$(x-x_0)^2+(y-y_0)^2=1.$$

If we make a “yx” plot of this function, it is a circle of radius 1 centered at (x_0, y_0).

(a) We may write the ideal gas law in terms of y and x as $yx = nrT$. Find an expression for the constant r.

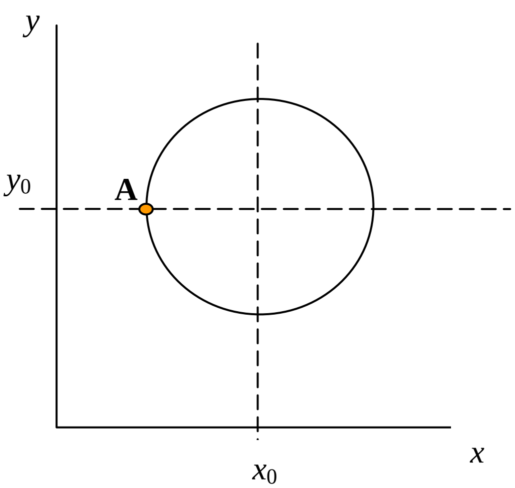
(b) Find an expression for work in terms of in integral involving dx.

(c) Using this expression, find the total work done by the engine in going once around the complete cycle.

(d) Write the adiabatic equation PV^{γ} = constant in terms of x and y. How would you evaluate the constant that appears in the new equation?

Problem 6.

Consider the engine described in Problem 1 and illustrated to the right. Start at point **A**. Consider this point to be at an angle of 180°. Using excel or a similar spreadsheet, find the following quantities.



(a) x and y values for one degree increments starting at 180° and going to – 180° . Note that there are 361 points. (The simplicity of finding points on the circle is the motivation behind the change of variables.)
(b) T for each point, using the ideal gas law from Problem 1.
(c) dx, dy, and dT for each interval. This can be done by simple subtraction. Note there will be 360 values now.
(d) Find the average values of each quantity at each point, *xave*, *yave*, and *Tave*. There will also 360 of these points.
(e) Find dW for each of the 360 points using the formula from Problem 1. Use the formula from Problem 1. Use *yave*, etc., in this calculation.
(f) Find dE_{int} for each point. Assume the gas is a monatomic ideal gas so you can determine the value of C_V.
(g) Find dQ for each point.
(h) Find dS for each point.

Problem 7.

Now let's find a few results for this cycle. Use the following values:

$P_0 = 2.00 \times 10^5$ Pa
P_R = P0/2
$V_0 = 2.00 \times 10^{-3}$ m^3
V_R = V0/2
n = 0.160 moles
Assume the gas is a monatomic ideal gas.

(a) Find the total work for one cycle. (=sum(A1:A10) is a useful formula to use in the spreadsheet.) See that this agrees with your answer from Problem 1.
(b) Find the total Q for one cycle. How should this compare to the total work?
(c) Find the total change in E_{int} for one cycle. Does this equal what you would expect?

(d) Find the total change in entropy for one cycle. Does this equal what you would expect?
(e) Find Q_{in}. Note that you must sum all the values of Q that are positive.
(f) Find the efficiency of the engine. How does compare to the efficiency of a Carnot cycle operating between the same maximum and minimum temperatures?

Note that the dividing points beween positive and negative Q are the points where $dQ = 0$. That is, these small segments of the circle (and only these segments) lie on adiabatic lines. If you want to go the extra mile, plot the adiabatic lines that are tangent to the circle. Your result of Problem 1(d) may be useful. Note that whenever an adiabatic line is tangent to the engine's path, Q changes from positive to negative and the path has to be broken at that point to separate the heat in from the heat out in an efficiency calculation.