Slide 1 - Title

Physics 121

Class 9

Slide 2 - Today

Today

- Projectile Motion
- Collisions in 2-Dimensions
- Velocity Addition and Relative Velocity in 2-D

Slide 3 - Schedule

Schedule

- HW 8 is due tomorrow and HW 9 on Friday, as usual.
- Quiz #3 is due Monday rather than Saturday.
- Lab #3 is up this week.

Slide 4 - Header 4

Test Reminders
A Few Details

- Testing Center (http://testing.byu.edu)
  - Ends Thursday at noon!!!
    - Wednesday 8am-9pm in at 10 pm
    - Thursday 8am-noon !!!!!! in at 10 pm
  - No late days!!
- Check the slides from Thursday for other details

Last Time

- Test Review
  - Where and When
  - Test Mechanics
  - Resources
  - What to Study
  - Practice Problems
- Position, Velocity, and Acceleration in 3-Dimensions
  - Each vector equation is really 3 equations in 1
  - Time is the common element

Equations Look the Same

\[
\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}
\]

\[
\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}
\]

\[
= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}
\]

\[
\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}
\]

\[
= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}
\]
Equations Look the Same

\[
a_x = \frac{dv_x}{dt} \quad v_x = \frac{dx}{dt} \\
a_y = \frac{dv_y}{dt} \quad v_y = \frac{dy}{dt} \\
a_z = \frac{dv_z}{dt} \quad v_z = \frac{dz}{dt}
\]
**Equations**

\[ x = x_0 + v_{0x}t \]
\[ y = y_0 + v_{0y}t - \frac{1}{2} g t^2 \quad \text{is up} \]
\[ v_y = v_{0y} - g t \]
\[ v_y^2 - v_{0y}^2 = -2g(y - y_0) \]

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**Problem**

A baseball is hit from an initial height of 1.60 m with a velocity of 62.0 m/s directed at an angle of 42.0 degrees with respect to the horizontal. What is its range?

Strategy:
1. Change the initial velocity into x and y components.
2. Use the x and y equations of motion.

One catch:
\[ t = 8.50, -0.0384 \] Which one is good?

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**Problem**

A baseball is hit from an initial height of 1.60 m with a velocity of 62.0 m/s directed at an angle of 42.0 degrees with respect to the horizontal. How high did it go?

2 1/2 methods:
1. No-t equation.
2. \( v \)-equation
3. 2 1/2. differentiate \( y \)-equation (\( dy/dt = 0 \) at top)
A Collision Problem

Two hockey pucks of masses m and M collide elastically. They initially have velocities \( v = v_i + v_j \) and \( v = V + V_j \), respectively. Write the equations you need to use to solve this problem using conservation of energy:

\[
me_i + MV_i = p_i = me_i' + MV_i',
\]
\[
me_f + MV_f = p_f = me_f' + MV_f',
\]
\[
\frac{1}{2}me_i^2 + \frac{1}{2}MV_i^2 = K = \frac{1}{2}me_i'^2 + \frac{1}{2}MV_i'^2
\]

How many unknowns are there?

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Another Collision Problem

Two hockey pucks of masses m and M collide elastically. They initially have velocities \( v = v_i + v_j \) and \( v = V + V_j \), respectively. Write the equations you need to use to solve this problem using conservation of energy. The small puck comes off the collision at an angle \( \theta \) measured from the x-axis.

Now how many unknowns are there?

This problem is doable, but we won't do it!
Put the problem in the cm

\[ m = 2 \quad M = 3 \quad v = -0.6i - 4.2j \quad v' = 0.4i + 2.8j \quad \theta = 65^\circ \]

In the cm frame, elastic collisions just change directions, not speeds!

How do we know that satisfies conservation of momentum and kinetic energy?

Recapitulation

1. Change everything to x and y components
2. Find the total momentum before the collision and divide by total mass (like totally inelastic problem! - this is the velocity of the cm
3. Subtract off the velocity of the cm from each velocity to get the velocities in the cm
4. Check momentum conservation in the cm
5. Find cm speeds before the collision - these are also the speeds after the collision
6. Using the cm angle, find the new cm velocities
7. Add the velocity of the cm to these to get the new lab velocities.
8. Check energy and momentum conservation.

Adding Velocities in 2-D

\[ \text{Speeds:} \quad w = 4.24 \quad W = 2.83 \quad (w's \text{ are cm}) \]

\[ \text{Angles:} \quad \theta = 65^\circ \text{ and } \theta = 180^\circ + 65^\circ \text{ (Must be back to back!)} \]

\[ w' = 1.79i + 3.85j \quad (\text{primes are after the collision}) \]

\[ W' = -1.20i - 2.56j \]
Crosswinds

https://www.youtube.com/watch?v=tw+lRTv6vJU

Same As 1-D but in 2-D

Velocity in lab:
\[ \vec{v} = v_x \hat{i} + v_y \hat{j} \]

Velocity of observer (river, air, etc frame):
\[ \vec{u} = u_x \hat{i} + u_y \hat{j} \]

Velocity with respect to the observer:
\[ \vec{w} = \vec{v} - \vec{u} = (v_x - u_x) \hat{i} + (v_y - u_y) \hat{j} = w_x \hat{i} + w_y \hat{j} \]

If you know the velocity in the observer's frame instead:
\[ \vec{v} = \vec{w} + \vec{u} = (w_x + u_x) \hat{i} + (w_y + u_y) \hat{j} = v_x \hat{i} + v_y \hat{j} \]

Problem

You have a large cattle ranch with a river running through it. The river is fairly calm, and flows to the west with a speed of 1.25 m/s. You’ve found from experience that the best way to cross the river directly to the north is to row towards a particular mountain in the distance. The mountain’s compass bearing is 230 degrees. If you rowed at the same speed in a lake, how fast would you be going?