Slide 1 - Title

Physics 121

Class 15

Slide 2 - Schedule

Schedule

- Quiz #5 due Saturday - covers HW 12 and 13

Slide 3 - Today

Last Time

- You saw how motion in a circle is mathematically similar to motion in a straight line.
- You learned that there is a centripetal acceleration (and force) and a tangential acceleration (and force) when objects move in circular paths.
- You learned that the magnitude of the centripetal force - whatever the real force is that keeps an object moving in a circular path -  is \( \frac{v^2}{r} \).
- You also learned a new way of dealing with motion as viewed from an accelerating reference frame; that is, how does a ball appear to fall in an accelerating car?

Slide 4 - Today

Today

- We will consider how concepts of angular velocity and acceleration apply to rotating objects.
- We will define torque and see that it is the angular analog of force.
- We will consider torques on particles and on rolling objects.
Problems from Last Time

Dropped Ball
You are in a car that is going down a straight stretch of freeway at a constant speed of 70 mph. What happens when you drop a ball?

Dropped Ball
You are in a car that is going down a straight stretch of freeway but now you're accelerating (increasing your speed). What happens?

Think of the car initially at rest and the speed of the car is $v = at$. If the ball is dropped at $t = 0$, what does it's motion look like from the road? What does the car's motion look like? What is the position of the ball as measured in the car?

One way of thinking about this problem is to treat the car as if it were at rest, but say there is a force $\vec{F}_{\text{friction}} = -ma_{\text{air}}$ acting on the ball as it falls.
Pendulum in a Car

A (very fast) car is accelerating at the rate of 0.8 m/s². A pendulum is attached to the roof of the car. What angle does the pendulum make, taking straight down as 0°?

\[ F_{\text{fictitious}} = -ma_{\text{car}} \]

Draw a “fake” free-body diagram with the fictitious force included.

What happens when the car goes around a corner?

Ball on a String

A ball on a string moves in a circular path in a vertical plane. Assume the speed is constant.

Draw free body diagrams for top, bottom, and an arbitrary angle.

What is the tension in each case?

Driving in Circles

An example from the text:

A car is driving on a circular road. If you know the (effective) coefficient of static friction for sideways motion (not forward), what is the maximum speed the car can attain?

Driving in Circles

An example from the text:

Repeat the last exercise, but assume the road is banked.
We define angular velocity and angular acceleration just like we did for an object traveling in a circular path.

\[ \omega = \frac{d\theta}{dt} \]
\[ \alpha = \frac{d\omega}{dt} \]

Constant \( \alpha \)

\[ \omega = \omega_i + \alpha t \]
\[ \theta = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \]
\[ \omega^2 - \omega_i^2 = 2\alpha (\theta - \theta_i) \]
True for “Chunks” of the Wheel

- We can define linear velocities (that is m/s, not rad/s) for a chunk of the wheel, but not for the wheel as a whole.

\[ s = r \theta \]
\[ v = r \omega \]
\[ a_{	ext{centripetal}} = \frac{v^2}{r} \]

Other Questions

- How do you get a wheel turning to start with? Does it involve force? work?
- A rotating wheel clearly has energy and some kind of inertia or momentum. How can we define those?
- Do wheels of the same mass but different distributions of mass respond to forces the same way?

-- Answers will come later.

Rolling Wheel

Roll a wheel across the floor. How is that different than a wheel in space?

Assume the wheel rolls without slipping.
Rolling Wheel

We can still define angular quantities the same way.

\[ \omega = \frac{d\theta}{dt} \]
\[ \alpha = \frac{d\omega}{dt} \]

Constant \( \alpha \)
\[ \omega = \omega_0 + \alpha t \]
\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \]
\[ \omega^2 - \omega_0^2 = 2\alpha (\theta - \theta_0) \]

Rolling Wheel

The linear velocity is related to the angular velocity by some familiar equations.

\[ x = r\theta \]
\[ v = r\omega \]
\[ a = r\alpha \]

Loosening a bolt

How would you apply a force on the “wrench” to loosen the bolt most easily? Why?
Loosening a bolt

What makes the torque larger?

- The force is greater
- The wrench is longer
- The force is perpendicular to the wrench

Torque on a Particle

The torque on a particle in space is:

\[ \tau = r F_{\text{tangential}} = r F \sin \theta \]

Torque on a Particle

A little math:

\[ \tau = r F_{\text{tangential}} = rm \omega_{\text{tangential}} = mr^2 \alpha = I \alpha \]

I is called the “moment of inertia” or “rotational inertia.” It is like mass for rotations.

Torque on an Object
Torque on an Object

- Draw the force vector with its tail at the point the force is applied. This is $F$.

- Redraw $F$ so the tails of the two vectors are together. The angle between the vectors is $\theta$.

\[ \tau = rF \sin \theta \]

Visualizing Torque

- Torque is the perpendicular component of force times $r$. This is the best way to think of torque.

\[ \tau = rF \sin \theta \]
**Visualizing Torque**

Torque is the whole force times the "moment arm" \( d = r \sin \theta \). Draw a line along \( F \) and make a line from the center of rotation that’s perpendicular to that line.

\[
\tau = F r \sin \theta
\]

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**Examples**

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**Downhill Bicycle**

A bicycle rolls from rest down an incline for 3.00 s with an acceleration of 1.50 m/s².

What is the angular acceleration of a bicycle tire if its radius is 30.0 cm?

What is the angular velocity of the tire at the end of the 3.00 s?

How many radians has the tire turned? How many revolutions?

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**Pulley with Mass**

A mass of 1.00 kg hangs from a string that goes around a pulley of radius 10.0 cm. The mass falls downward with an acceleration of \( g/2 \).

What is the torque on the pulley?

What is the angular acceleration of the pulley?