Today

- You will see how motion in a circle is mathematically similar to motion in a straight line.
- You will learn that there is a centripetal acceleration (and force) and a tangential acceleration (and force) when objects move in circular paths.
- You will learn that the magnitude of the centripetal force - whatever the real force is that keeps an object moving in a circular path - is $mv^2/r$.
- You will also learn a new way of dealing with motion as viewed from an accelerating reference frame; that is, how does a ball appear to fall in an accelerating car?

Schedule

- Homework Wednesday (12 & 13) and Friday (14)
- Quiz Saturday
- Lab due Monday

Last Time

- Define energy
- Define potential energy
- Explain how energy is conserved - and isn't conserved
- Work some problems involving conservation of mechanical energy
- Explain how potential energy is related to work and to force
- Draw energy bar graphs
- Draw and use potential energy graphs
Slide 5 - Circular Motion

Circular Motion

Slide 6 - Particle Moving in a Circle

- Planets (almost)
- Satellites
- Ball on a string
- Conical pendulum
- Car on a circular track
- Charged particles in a constant magnetic field

Slide 7 - Acceleration at Constant Speed

A man is running around a circular track. At one instant, he is going west. A time Δt later, he is going south. What is the direction of his average acceleration?

Slide 8 - Acceleration at Constant Speed

\[ \vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \]

The acceleration vector points to the center of the circle.
**Centripetal Acceleration**

For circular motion at constant speed:
- the instantaneous acceleration is always toward the center of the circle
- the acceleration is always perpendicular to the velocity
- the acceleration changes direction without changing speed
- the acceleration must equal

\[
\vec{a}_c = -\frac{v^2}{r} \hat{r}
\]

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**Centripetal Force**

\[
\vec{F}_c = m\vec{a}_c = -\frac{mv^2}{r} \hat{r}
\]

- This is the centripetal force
- You never put it on a free body diagram
- If tension holds a ball in a circular path, tension goes on the free body diagram. Tension is the “real force” and if the speed is constant

\[
\vec{T} = -\frac{mv^2}{r} \hat{r}
\]

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**Centripetal vs Centrifugal**

- Centripetal means “center seeking”
- Centripetal force is whatever real force that keeps an object moving in a circular path
- Centrifugal means “center fleeing”
- Centrifugal force is the force you feel throwing you outward as a car turns or a merry-go-round rotates.
- Centrifugal force is a “fictitious” force because it’s just the affect of your car accelerating under you.
- Centrifugal force is real in that you accelerate with respect to your car.
Circular Motion the Easy Way

For uniform linear motion, \( x \) increases linearly in time:
\[
x = vt
\]
For uniform circular motion, \( \theta \) increases linearly in time:
\[
\theta = \omega t
\]
\( \omega \) (omega) is the “angular velocity”

Similarly, linear and angular velocity are related:
\[
v = r\omega \quad (\omega \text{ in radians/s})
\]

Linear and Angular Motion

\[
v = \frac{dx}{dt} \quad x = \theta \quad \omega = \frac{d\theta}{dt}
\]
\[
a = \frac{dv}{dt} \quad a = \alpha \quad \alpha = \frac{d\omega}{dt}
\]
Constant \( \alpha \)
\[
v = v_0 + at \quad \omega = \omega_0 + \frac{1}{2} \alpha t^2 \quad \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2
\]
\[
x = x_0 + v_0 t + \frac{1}{2} \alpha t^2 \quad \omega^2 - \omega_0^2 = 2\alpha (\theta - \theta_0)
\]

Two Accelerations

\( \alpha \) is the “angular acceleration.” It tells how the angular velocity changes in time.
\[
a = r\alpha \quad \text{is the “tangential acceleration.” It tells how the “orbital” speed of the object changes in time. Its direction is tangent to the circle.}
\]
\( a \), the centripetal acceleration is independent of the tangential acceleration and always equals \( \frac{v^2}{r} \) for circular motion, even if \( v \) is changing.
Two Accelerations

\[ \ddot{a}_{\text{eff}} = \text{the sum of a radial part and a part that is parallel to } \ddot{y}. \]

Units

\[ \theta \text{ is in radians -- radians really have no units.} \]
\[ s = r \theta \text{ Note that } s \text{ and } r \text{ are both in meters.} \]
\[ \omega \text{ is in radians/s or just s}^{-1} \]
\[ \alpha \text{ is in radians/s or just s}^{-2} \]

\[ m \frac{v^2}{r} \]
\[ \theta \text{ is usually measured with respect to the x-axis and increases in the counter-clockwise sense.} \]
\[ \text{If the angular velocity is constant} \]
\[ \dot{\theta} = \omega t \]
\[ \vec{r} = \vec{r} \sin \omega t \hat{i} + \vec{r} \cos \omega t \hat{j} \]
\[ \mathbf{v} = -v \sin \theta \mathbf{i} + v \cos \theta \mathbf{j} \]

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = -v \omega \cos \theta \mathbf{i} - v \omega \sin \theta \mathbf{j} \]

\[ = -v \omega (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) \]

\[ = -v \omega \mathbf{r} \]

\[ a = v \omega = \frac{v^2}{r} \]

**Examples**

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**Conical Pendulum**

What forces are in a free-body diagram?

What do you know about the vertical component of tension?

What do you know about the horizontal component of tension?

What is the angular speed?

What is the speed of the ball?

What is the direction of the acceleration?
Ball on a String

A ball on a string moves in a circular path in a vertical plane. Assume the speed is constant.

Draw free body diagrams for top, bottom, and an arbitrary angle.

What is the tension in each case?

Driving in Circles

An example from the text:

A car is driving on a circular road. If you know the (effective) coefficient of static friction for sideways motion (not forward), what is the maximum speed the car can attain?

Driving in Circles

An example from the text:

Repeat the last exercise, but assume the car is road is banked.

Accelerating Frames
Dropped Ball

You are in a car that is going down a straight stretch of freeway at a constant speed of 70 mph. What happens when you drop a ball?

Think of the car initially at rest and the speed of the car is \( v = a \). If the ball is dropped at \( t = 0 \), what does its motion look like from the road? What does the car's motion look like? What is the position of the ball as measured in the car?

Dropped Ball

You are in a car that is going down a straight stretch of freeway but now you're accelerating (increasing your speed). What happens?

One way of thinking about this problem is to treat the car as if it were at rest, but say there is a force

\[
\vec{F}_{\text{fictitious}} = -m\vec{a}_{\text{car}}
\]

acting on the ball as it falls.

Pendulum in a Car

A (very fast) car is accelerating at the rate of 0.8 m/s². A pendulum is attached to the roof of the car. What angle does the pendulum make, taking straight down as 0°?

\[
\vec{F}_{\text{fictitious}} = -m\vec{a}_{\text{car}}
\]

Draw a “fake” free-body diagram with the fictitious force included.

What happens when the car goes around a corner?
End