Homework Assignment #6

Do the following problems:
7-3, 7-4(a)*, 7-6*, 7-9, 7-10*, 7-12*

and the following problem:

Consider a collection of non-interacting particles distributed at $t = 0$ according to the 1-d distribution function

$$f(x, v) = \frac{N}{\pi a} \sqrt{\frac{m}{2T}} \exp^{-\frac{mv^2}{2T}} \exp^{-\frac{x^2}{a^2}}$$

Since the particles are non-interacting and there are no external forces, the distribution function can be written down for all later times without the use of the Vlasov equation. In particular, the Vlasov equation implies that $f(x, v) = f(x_0, v_0)$, where $x_0$ and $v_0$ are the initial values of $x$ and $v$.

(a) Use your knowledge of how $x$ and $v$ depend on time in this case to write down the distribution function for all later times $t$.

(b) By completing the square in the exponent of the distribution function obtained in part (a), obtain the following values by inspection:

- The temperature, $T^*$, as a function of time.
- The fluid velocity as a function of time and space.
- The spatial width of the distribution, $a^*$, as a function of time.

(c) From the results of (b) plus the fact that

$$\int_{-\infty}^{\infty} \exp^{-A(v-v_0)^2} dv = \sqrt{\frac{\pi}{A}}$$

to find $n(x, t)$. (It might be easiest to express it in terms of $a^*$.)

Note: The problems marked with an asterisk are worth 10 points each, as is the additional problem. All others are worth 5 points.