

Physics 430, Winter 2011
Take-Home Problem #2
3 March 2011
due 10 March 2011

Solve the following problems in Matlab. Email a copy of your scripts to Dr. Peterson (bryan.peterson@byu.edu) by midnight on the due date.

This exam is open notes (code, lab manual, matlab book, etc.) but is closed lab partner and closed lab TA. You are expected to do your own work but you are allowed to reuse appropriate code from previous exercises.

Please also include a note indicating approximately how long you spent on this exam.

1. Consider the wave equation

$$\frac{\partial^2 y}{\partial t^2} - e^x \frac{\partial^2 y}{\partial x^2} = 0$$

on the interval $x \in [0, 2]$ with boundary conditions $y(x, t) = 0$ at $x = 0$ and $\partial y / \partial x = 0$ at $x = 2$. Use a cell-centered grid for your solutions.

Animate this wave equation using staggered leapfrog with initial conditions $y(x, 0) = \sin(3\pi x/4)$ and $\partial y / \partial t = 0$. Have your code create a graph of $y(x)$ within 1/2 timestep of the time $t = 4$ and pause so that I can examine the graph.

2. Next, consider the diffusion equation

$$\frac{\partial T(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(D(x) \frac{\partial T(x, t)}{\partial x} \right)$$

on the interval $x \in [-1, 1]$ with boundary conditions $T(x, t) = 0$ at $x = -1$ and $\partial T / \partial x = 0$ at $x = 1$. I would recommend a cell-centered grid (because you have a Neumann boundary condition at $x = 1$). The diffusion coefficient is given by

$$D(x) = 2e^{x^2}$$

and the initial temperature is

$$T(x, 0) = 1 + x + 2 \sin \left(\frac{(x + 1) * \pi}{1.4} \right) .$$

Find the solution of this equation for $t = 0$ to 1. Create a surface plot of the solution similar to the figures in Lab 7 in the manual. The axes on the surface plot should be properly labeled and you should include the initial temperature at $t = 0$.

Using more than about 40 or 50 points in each direction on the surface plot will result in a "less useful" view of the solution.