Driven harmonic oscillators:

\[ \ddot{x} + 2\beta \dot{x} + \omega_0^2 x = f(t) = F(t)/m \]

For sinusoidally driven oscillators:

- The general solution combines the solution to the damped, undriven oscillator + a solution to the driven oscillator.
- The particular solution to the driven oscillator is the steady-state solution, the solution to the damped oscillator is the transient solution.
- We usually take a complex solution and keep the real part.

The basic steady-state solution is

\[ x(t) = A \cos(\omega t - \delta) \]

where

\[ A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} \]
\[ \tan(\delta) = \frac{2\beta\omega}{\omega_0^2 - \omega^2} \]

(you should have the general solution memorized, but you don’t need to remember the expressions for the coefficients)

Be able to sketch \( A^2(\omega) \) and \( \delta(\omega) \).

The full width at half max (FWHM) of the \( A^2 \) curve is approximately \( 2\beta \).

Q (quality factor)

\[ Q = \frac{\omega_0}{2\beta} = \pi \frac{\text{decay time}}{\text{period}} \]

Work problems 5.27 and 5.43 from the text (pp. 210 and 212). This may be submitted on paper or as a document (pdf preferred) attached to your email. NOTE: problem 5.43 includes the word “estimate.” That means you do NOT have to do a detailed analysis of where each of the men sits and his relationship to each of the wheels. It is not quite at the level of “guess” as recommended by Bones in Star Trek IV when Spock needed information to get back from 20th Century Earth – but it is close.
1. A 5.00 g mass made of iron hangs vertically from a spring of spring constant 6.00 N/m. It is driven sinusoidally by an electromagnet that provides a maximum force 0.0005 N on the mass. The system is characterized by a damping constant of $\beta = 2$.

(a) Solve the differential equation for the system with the driving frequency $\omega_d$ left as a variable. Plot the early and late behavior of the system for three driving frequencies: $2\omega_0$, $\omega_0/2$, and $\omega_0$. Take the initial conditions to be $x(0) = 0$ m and $v(0) = 0$ m/s. From the graphs estimate the amplitude of the oscillation after the initial transients have died away. At which driving frequency does the maximum amplitude occur?

(b) From the solution of the differential equation above, find the part that corresponds to the steady-state solution and cut and paste it into a new function. Plot this function and find the maximum amplitude of the solution from the equation. Remember to use the driving frequency that is expected to give the maximum steady-state amplitude.

(c) Using the equation for $A$ derived in the text (Eq. 5.64), plot $A$ as a function of the driving frequency. Call the driving frequency $\omega_d$, as we have a fixed value for $\omega$ already defined. Be sure that the values of $A$ you obtained for the three frequencies in part (a) above are consistent with this curve.

(d) Now plot $A^2$ as a function of the driving frequency. Estimate the FWHM (full width at half maximum) of this curve graphically. (Note: this can be done most easily by plotting the curve for values of $A^2$ ranging from $A_0^2/2$ to $A_0^2$.) Compare this number with $2/\beta$. Determine the quality factor, $Q$ of the oscillator.