Conservation of energy can be used in two ways to derive equations of motion for a system:

A. $T + U = E$. For free fall, for example, this leads to:

\[ \frac{1}{2} m \dot{x}^2 + m g x = E \]
\[ x^2 = \frac{2(E - m g x)}{m} \]

B. $\dot{T} + \dot{U} = 0$.

Be careful that you correctly identify the total kinetic energy and potential energy of the system.

1. A small bar of soap with mass $m = 125$ g is placed in a bowl of hemispherical shape. The soap can slide frictionlessly on the surface of the bowl. The radius of curvature of the bowl is $R = 50.0$ cm. Use a coordinate system where the $z$ axis points upward. (Thus, when the soap is in the bottom of the bowl $\theta = \pi$.) The soap has no component of velocity in the $\phi$ direction; that is, it slides only “up” and “down” the bowl. Let the bar of soap be initially at an angle of $\theta = 5\pi/6$ (30° up from the bottom of the bowl), and be moving up the bowl with a velocity of 2.00 m/s.

   (a) Write down $T(v_\theta)$ and $U(\theta)$.
   (b) Algebraically (not graphically) find the classical turning points.
   (c) Make a plot showing $U(\theta)$ and $E$.

2. Write down equations for $T$ and $U$ for the same situation as in problem 9.1. This time, however, make the following substitutions so that we can differentiate these functions with respect to time:

   in $T$ change $v_\theta$ to $R*theta'[t]$ or $R*D[theta[t],t]$
   in $U$ change $theta$ to $theta[t]$

   (a) Use the equation $\dot{T} + \dot{U} = 0$ to solve for $theta[t]$.
   (b) Plot theta in degrees vs. t.
   (c) Do the classical turning points match the values found in problem 9.1? Describe the motion of the soap.