Several important concepts from the last couple of class periods:

A. We can define a potential energy only under some quite restrictive conditions:
   a. The force must be a function of position only. It cannot have velocity or time
dependence. It can be a constant.
   b. The curl of the force must be zero. If this is true the line integral around any
closed path is zero.

B. The gradient
   a. operates on a scalar field and gives us a vector field.
   b. tells how much and in what direction the scalar field increases most rapidly.
   c. in Cartesian coordinates can be written
      \[ \nabla \psi = \frac{\partial \psi}{\partial x} \hat{x} + \frac{\partial \psi}{\partial y} \hat{y} + \frac{\partial \psi}{\partial z} \hat{z}. \]

C. \[ \vec{F} = -\nabla U \] if a potential energy exists.

D. The curl:
   \[ \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \]
   \[ (\nabla \times \vec{A})_x = \lim_{\Delta A \to 0} \frac{\int_{\text{path in y-z plane}} \vec{A} \cdot d\vec{l}}{\Delta A} \]

1. Using both Cartesian and spherical coordinates, show that a central, inverse square
force is conservative. (Hint, show that the force has zero curl.)
   Use the form \( F = \alpha/r^2 \) for the magnitude of the force.

2. In cylindrical coordinates the magnitude of a force is given by the expression \( F = \alpha/r^2 \)
   where \( \alpha \) is a constant. Remember that \( r \) is the cylindrical coordinate. Is this a
   conservative force if the direction of the force is radial? What about tangential?

3. Find the force for each of the following potentials. \( \alpha \) is a constant.
   (a) \( U = -\alpha/r^3 \) in spherical coordinates.
   (b) \( U = \alpha \sin(\theta) \) in cylindrical coordinates.