By now you are probably getting fairly proficient with solving problems using Mathematica. If not, you should invent a few differential equations and try working through solutions, plotting results, etc. (If you are new to Mathematica you will find that it has a few idiosyncrasies. Sometimes it just refuses to do what you want it to do. Most often that is because you have a typographical error in the command. Check the syntax and variable names carefully.)

So, for Lorentz forces you should note:

a) Mathematica will do cross products, but in Cartesian coordinates it is usually just as easy to do them by hand and just enter the results.

b) The Lorentz force does not change the speed of an object, only the direction of motion.

c) In uniform fields, charged particles spiral with the cyclotron frequency and radius given by the equations:

\[ \omega_c = \frac{qB}{m} \]

\[ r_c = \frac{p}{qB} \]

Review sections 2.5 through 2.7 so you understand how to derive these equations.

1. The magnetic field of a wire is given by the expression \( B = \frac{0.305}{r} \) where \( r \), the distance from the center of the wire, is in meters and \( B \) is in Tesla. The wire is on the \( z \)-axis and current flows in the \( +z \) direction. Recall that a right-hand rule will give you the direction of the field lines around the wire. A charge of \(+2.43 \, \text{mC}\) and mass \(0.100 \, \text{g}\) moves under the influence of the magnetic field of the wire. The initial conditions are:

\[ x(0) = +0.100 \, \text{m}, y(0) = 0, z(0) = 0 \]

\[ \dot{x}(0) = 0, \dot{y}(0) = 0, \dot{z}(0) = +1.00 \, \text{m/s} \]

Plot the motion of the particle over the time interval 0 to 1 second.

2. Th motion observed in problem 4.1 is sometimes referred to as the “grad-B” drift. This drift describes the motion of the “guiding center,” or the motion of the apparent center of rotation of the particle. The equation that describes this motion is

\[ \vec{v}_d = \frac{\epsilon}{qB} \frac{\vec{B} \times \nabla B}{B^2} \]
where $\epsilon_\perp = \frac{mv_\perp^2}{2}$ is the kinetic energy due to the component of motion perpendicular to the magnetic field. Since $\vec{B}$ in this problem is best described in cylindrical coordinates, the gradient operator is given by

$$\nabla B = \frac{\partial B}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial B}{\partial \theta} \hat{\theta} + \frac{\partial B}{\partial z} \hat{z}.$$  

Determine whether the drift velocity (both direction and magnitude) is consistent with the equation for the drift velocity (use your graphs from part 4.1 to estimate the drift velocity).