

I. AN INTRODUCTION TO EXPERIMENTATION

Overview

Before you are given a description of any detailed experiments, let's review the activities involved in any experimental laboratory work in physics. These are important to each of the weekly laboratory outlines so they should be well understood by the student.

Seven essential elements involved in any experimental study are:

1. Identify and isolate in your mind questions that need to be answered and can be resolved through the experimental study.
2. Design apparatus and/or formulate procedures to make measurements relating to a phenomenon or system you desire to understand.
3. Make measurements and record data either numerically or graphically.
4. Analyze, organize, summarize, correlate, and in any other way study the data to find all the direct information as well as any hidden insights the data may contain.
5. Ascertain the accuracy or reliability with which you made the measurements. Identify sources of error and make corrections for known extraneous effects.
6. Interpret and evaluate the results of the observations as they relate to the questions identified previous to the design of the experiment.
7. Report the significant conclusions.

Each of these items will be briefly discussed below as an encouragement for you to think seriously about the experimental process. To be successful, every experimenter must develop skills in each of these areas. Your ability to perform these functions will increase throughout your professional career.

Identification of a Problem

Before initiating an experimental study, it is of importance to clarify precisely what questions you want to answer. It is not enough to say, "I want to see what happens." Some experimental work simply answers the question, "How does one parameter vary with a second parameter?" Other experiments are designed to simply improve the accuracy of data in order to prove or disprove some theoretical prediction or verify some functional relationship. The most meaningful experiments, however, are guided by theoretical considerations. As the experiment is formulated, the good experimenter will have in his mind one or more anticipated possible results based on alternative

explanations. The crucial factor in identifying questions is to be as specific as possible in formulating them.

Design of the Experiment

You must next identify what experimental information and data are required to answer the questions posed in as unambiguous a manner as possible. You must decide precisely what you want to measure and the results you expect.

Much of this process is included in the manual for parts of the outlined experiments. However, there are other parts of the experiments for which you must consider the design of the experiment yourself. When planning your own measurements, think through various approaches before you begin taking data. Some approaches are easier than others; some are more accurate than others. Try to be creative in your approach to the problem.

Experimental Measurement

Experimental measurements fall into two broad categories:

1. Measurement of a specific physical quantity that can be specified by one or more numbers.
2. Measurement of a functional relationship in which a physical quantity varies with and depends upon one or more other physical quantities.

The second type of measurement is simply a set of measurements of the first kind. The fact that a functional relationship, known or unknown, is implied makes the analysis more difficult, more intriguing, and more significant. Most of science and engineering is associated with such functional relationships. For example, the fundamental laws of physics are simply statements of functional relationships that have been measured and verified. The understanding of any phenomenon of science is a direct consequence of understanding such functional relationships.

Note also that a functional relationship does not necessarily imply a mathematical equation. A function can be represented by a graph taken directly from experimental measurements.

Analysis of the Data

For a measurement of a single value, the only analysis of the data possible is the evaluation and determination of an estimated accuracy for the measurement. For measurements involving functional relationships, the analysis is more rich and interesting. Even when studying physical quantities that are functions of only one variable, you can divide the analysis into several steps as follows:

1. Display the data in either graphical or tabular form.
2. Search for mathematical equations that “fit” or “describe” the experimentally observed graphical function. Such equations are known as empirical equations in contrast to equations derived from fundamental laws. Note: Many of the fundamental laws

themselves are simply empirical equations that have been “canonized” (accepted by the scientific community).

3. Compare the data with theoretical predictions that are derived from fundamental laws or postulates. This aspect of analysis may involve your creating a theory to explain your results. To formulate a theory, you must propose a simple “model” and then apply known principles in an attempt to predict the data measured.

An analysis at the level of step (1) has value only as a predictive tool. The data so displayed allow you to predict the outcome of other measurements by interpolations and extrapolation from the data. The analysis at the level of step (2) is the heart of data reduction and analysis, for it is a search for mathematical relationships that will give insight into the fundamental physical laws that are operating. Details on how to find a mathematical equation that fits specific data are given in a separate section later.

When you have completed steps (1) and (2), you will find the real understanding and meaning behind the experiment in step (3). Only when you have made some comparison between what you actually measured and what you anticipated or postulated based on some model (imperfect as it may be), do you gain insight and understanding of the phenomenon involved. Measurement of numbers with no thoughts or ideas behind them is not the work of a scientist but rather the work of a technician. You may note that step (1) above is easy, step (2) is more demanding, and step (3) requires prior background and understanding.

A fundamental postulate of modern physical science is that nature can be described in mathematical terms and that the simplest mathematical form that adequately describes the phenomenon is the preferred analysis. The derivation from fundamental principles of mathematical relationships that have been previously discovered empirically provides challenges and goals for significant theoretical analysis. In rare cases, new empirical relationships form the basis for new laws of science.

Reliability and Error Evaluation

Any measurement is subject to both uncertainty and error. The evaluation of errors is required to validate the arguments in the analysis of steps (2) and (3) above and often precedes and/or is mingled with the analysis. All empirical equations will have undetermined parameters. The accuracy to which these parameters can be determined will always be in question and constitutes a major effort in data reduction and error evaluation. In many experiments there will be known extraneous effects for which you can compensate by making corrections to the data, so the corrected data can be analyzed as if the extraneous effect were not present. When you find differences between theory and experiment, you must ask whether the errors are in the theory, in the apparatus, or in the data-taking process.

When considering the accuracy of any measurement, you may think of the uncertainty associated with the measurement in one of three ways: instrumental uncertainty, statistical uncertainty, or systematic error. These types of uncertainty will be discussed below.

1. Instrumental Uncertainty

If you were to measure the length of a table with a meter stick, you could give an accurate measurement to the nearest millimeter or perhaps half a millimeter, but you clearly couldn't measure the length to nearest micron. Similarly, any instrument will have some inherent limitation to its precision.

2. Statistical Errors

If you were to measure the period of a pendulum with a stopwatch one hundred times, you wouldn't necessarily expect to get the same measurement each time. Because of small differences in the way the measurement was carried out, the results would differ from each other in a random manner. These measurements would scatter around the average value, and the average value would be more correct than any single measurement. You can analyze such random errors by using statistical techniques summarized in a later section.

3. Systematic Errors

Even if the stopwatch were to give the same reading each time, you would not know *a priori* if these measurements were really accurate. In order to ascertain their accuracy you would have to check the stopwatch against a known standard. Whenever precision measurements are made, proper calibration against known standards is absolutely essential. Errors, which result due to poor calibration and/or poor experimental technique, are termed systematic errors. Systematic errors are often harder to find and handle, but can be significant.

Even when we take into consideration all of the sources of uncertainty in a measurement, the results at times do not agree with theory. We may loosely speak of this as experimental error; however, if the experiment was properly done, the error is really in our thinking about the experiment. For example, in physics one speaks of massless ropes, point masses, point charges, perfect electrical and thermal insulators, frictionless surfaces, etc. Such idealizations serve a very important role in developing the theoretical structure of science, but they are most important in the initial development, and must be carefully scrutinized if you expect precise agreement with experiment. After you study the elementary treatment of the idealized situation, you must develop a more meaningful scientific model in which you analyze extended mass, extended charge, heat loss through conduction, surfaces with friction, and other more realistic conditions.

Interpretation of the Results

After having completed the experiment, reduced and analyzed the data, and evaluated the accuracy of the measurements, you must perform the most difficult task of the total effort. You must relate the insights and understandings gained to other scientific knowledge previously in existence and evaluate how these new insights change your previous thinking. This process involves relating your results to previous theoretical treatments, postulates, or other known data. It might include a comparison with previous predictions or previously obtained data, and the question of accuracy will be of prime importance.

Reporting of Results

Experimental work is not finished until it is reported to those for whom the work is of importance. For students that may be only the instructor. For industrial scientists and engineers, it may be the immediate supervisor, and for the professional scientist, the results of experimentation must be reported by publishing results in scientific journals. The reporting of the results will generally involve drawings of apparatus, tables and graphs of data; as well as mathematical analysis, verbal description and argumentation. It is difficult to make poor data and analysis sound good. Furthermore, a poorly prepared report will give a shabby impression of even a very good experiment and analysis. The process of presenting results involves a selection of the significant ideas, the condensation of these ideas into concepts, and the elimination of irrelevant data and discussion.

Before you begin experimentation, it is highly beneficial for you to realize the interwoven nature of experimental design, data collection, data reduction and analysis, and error evaluation. Success in an experimental laboratory will depend upon this understanding.

II. SUMMARY OF ELEMENTARY STATISTICAL ANALYSIS

Multiple Measurements and Mean Values

Consider the following example of using statistical methods to evaluate the uncertainties associated with random errors. Think of making several measurements of the distance between two points along a straight line. These measurements, which could be made by the same individual or by several different people with different measuring instruments or various procedures, may yield a variety of values of that single distance. If you assume the errors in the individual measurements could increase or decrease the determined length with equal probability (that is, if the errors are random), the average of all N measured values would be the best estimate of the true value. If systematic errors exist, the errors are really not random, and you must separately consider such errors. Proceeding with assumed random errors, designate the i th measured distance as x_i and the mean or average of these measured values by the symbol \bar{x} . In mathematical form:

$$\bar{x} = \frac{(x_1 + x_2 + x_3 + \dots + x_N)}{N} = \frac{\sum x_i}{N}$$

where the sum is understood to run from 1 to N .

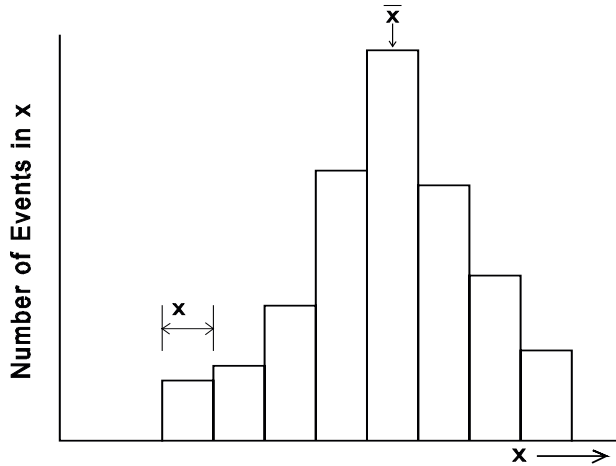
Now consider a second example of the use of statistical analysis in experimental studies. Think of measuring the diameters of N individual hairs in your head. If you make precise measurements on several individual hairs, these measurements will not yield the same value but rather a distribution of values because each hair is actually different in size. If you designate each individual measurement as x_i , you can calculate a mean \bar{x} as above. The value \bar{x} , however, now represents your best estimate of the size of any specific hair of your head selected at random; however, there is no such thing as a “true” value.

In both examples given, and in fact in all measurements in which you use statistical methods, the amount of variance or scatter in the measured values as well as the determined mean value, has great significance. The variance is an indication of the expected deviation of a single measurement from the mean value \bar{x} .

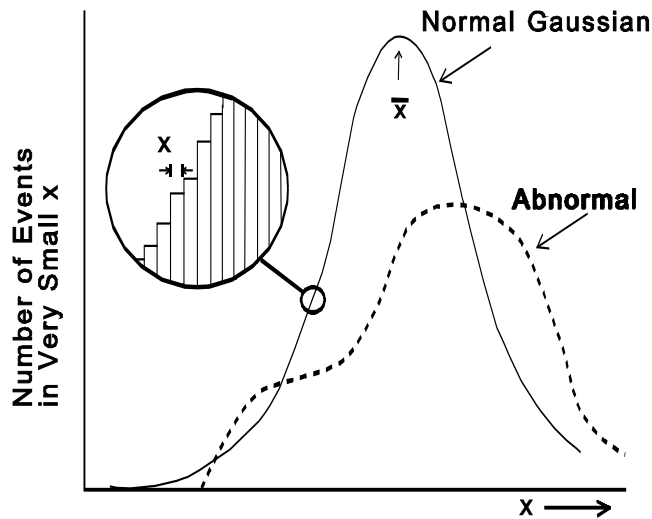
Histograms and Probability Distributions

When you make a reasonably large number N of measurements, you obtain a conceptual understanding of the size of the scatter and check for the existence of any abnormalities in the overall distribution by making a so-called histogram of the measurements. The histogram is a bar graph obtained by grouping together all measurements that fall within predetermined small intervals of the variable, Δx . You must judiciously select the size of the interval, Δx , such that a suitably large number of measurements fall in each interval. You can best understand the histogram by studying the example shown in Fig. 1. There must be several intervals in the histogram, but also there must be several measurements in each interval. As you make more and more measurements, the size of the intervals can be reduced to retain approximately the same number of counts in each interval. As

this process continues, the histogram becomes more meaningful.



Now consider making an enormously large number of measurements of a value x under similar conditions. You could construct a histogram with very small intervals, Δx , and a large number of measurements will still fall in each interval. Now the tops of the bars of the histogram will appear almost as a smooth curve. If you were to increase N indefinitely, a smooth curve called the parent probability distribution would result. Many physical processes that have natural random variations exhibit a probability distribution that is a very symmetrical, bell-shaped distribution known as the Gaussian or normal distribution. Fig. 2 shows a Gaussian distribution and also an abnormal probability distribution for comparison. The parent distribution provides the most detailed information available about a statistical process. In practice, however, the distribution is generally not known unless a very large number of measurements have been made. An experimental histogram represents the best approximation to the distribution that can be obtained from the data. Faced with the lack of knowledge of the parent distribution, it is common to assume in experimental work that random errors follow a Gaussian distribution.



Standard Deviation and Probable Error

The standard deviation is a single numerical value that quantitatively expresses the spread, scatter, or variance of the measurements around the average value \bar{x} . You can calculate the standard deviation for a given set of N measurements and can also relate this value to the “width” of the distribution peak. To develop the ideas related to the standard deviation, return to the example of N individual x_i measurements. You can easily calculate \bar{x} and then the individual deviation $d_i = \bar{x} - x_i$ of each measurement from the average. Some numerical measure of the average deviation is desirable, but the average of the deviations d_i is precisely zero since the meaning of the average implies an equal distribution of values above and below it. It is possible to calculate the average of the absolute values of the deviations. This approach gives rise to the definition of probable error

$$P.E. = \frac{|\bar{x} - x_1| + |\bar{x} - x_2| + |\bar{x} - x_3| + \dots + \mathbf{K}}{N} = \frac{\sum |d_i|}{N}.$$

However, absolute values are mathematically cumbersome functions because the first derivative is discontinuous. A more common practice, therefore, is to square the deviations, average the squares, and then take the square root of the average. This process yields the root-mean-square or standard deviation, σ_N :

$$\sigma_N = \sqrt{\frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + (\bar{x} - x_3)^2 + \dots + \mathbf{K}}{N}} = \sqrt{\frac{\sum d_i^2}{N}}.$$

Thus we see that a small standard deviation implies that the measurements tend to be close to the mean value. If all the measurements were identical, the standard deviation would be zero. Thus the standard deviation is a good estimate of the uncertainty associated with a series of measurements. For a set of measurements taken on items that have an actual variation of the variable within the group, as in the example of hairs given above, σ_N is appropriate. However, let's consider measurements of a single item in which the variations are associated with statistical errors, such as the measurement of a fixed length. If we take a single measurement of a length, then $N = 1$ and $\sigma_N = 0$. This implies one measurement is perfect and if more are taken uncertainty is introduced. Clearly that makes no sense when we are taking multiple measurements of a single object. In this case the appropriate standard deviation is σ_{N-1} , which is defined as:

$$\sigma_{N-1} = \sqrt{\frac{\sum d_i^2}{N-1}}.$$

The uncertainty associated with a single measurement is therefore undefined.

Whether or not the parent distribution is Gaussian, the standard deviation can be defined by the above equations for any set of measurements; however, the standard deviation is easily interpreted in terms of uncertainty when the distribution is Gaussian.

In order to proceed in this discussion, the assumption is made that the deviations d_i are Gaussian in nature. In other words, it is assumed that a very, very large number of measurements would yield deviations that follow a Gaussian distribution and that the average \bar{x} of all these measurements is the desired true value. In most measurements there is no way to justify such an

assumption without making a very large number of independent measurements and determining the actual probability distribution. Nevertheless, it is common to make this assumption if there is no reason to believe that the deviations are not Gaussian.

Listed below are some results that follow when you assume the distribution to be Gaussian. These results, which are proved in formal statistics courses, are very easy to use and have great utility in evaluating experimental data. You must realize, however, that the associated predictions are only valid statistically if the parent distribution is truly a Gaussian distribution.

1. If a large set of measurements are taken, 68.3 percent of the x_i values lie within the range $\bar{x} - \sigma$ to $\bar{x} + \sigma$, and 31.7 percent lie outside this range. This result implies that a single measurement is within σ of the true value 68.3 percent of the time.
2. It can be shown that the probable error $P.E. = 0.674\sigma$. This relationship implies that if you calculate σ , you immediately know the probable error. The probable error has the very simple interpretation that 50 percent of the \bar{x} values are within the range $\bar{x} - P.E.$ to $\bar{x} + P.E.$ Thus, if you were to make a single measurement, you could have a 50 percent confidence that this measured value was only one $P.E.$ away from the true value.
3. The Gaussian distribution also has the property that 95.5 percent of the x_i values are within the range $\bar{x} - 2\sigma$ to $\bar{x} + 2\sigma$, and 99.7 percent of the x_i values are within the range $\bar{x} - 3\sigma$ to $\bar{x} + 3\sigma$. In simpler terms, it is commonly stated that you can have a 95.5 percent confidence limit that a single measurement will be within 2σ of the true value or a 99.7 percent confidence that it will be within 3σ of the true value.

Quoting Experimental Uncertainties

In reporting experimental results, data are usually listed with a numerical value for the uncertainty. For example, the best value of the mass of an electron currently is $(5.48579902 \pm 0.00000013) \times 10^{-4}$ atomic mass units. Since uncertainty can be defined in many ways, how do we know what number to assign?

For the purposes of most analyses, and in particular for the purposes of this class, the rules for assigning uncertainty follow:

1. For statistical uncertainty, use one standard deviation (unless otherwise specified).
2. For instrumental uncertainty, estimate the uncertainty as well as you can. Although this uncertainty isn't strictly Gaussian, it is usually treated as if it were Gaussian in subsequent error analysis.
3. For known systematic errors, the data should be appropriately corrected and no systematic uncertainty quoted.

III. PROPAGATION OF ERRORS DUE TO MATHEMATICAL MANIPULATIONS

You may often desire to calculate a quantity mathematically from other values that have been obtained experimentally. Each of these experimental values will have associated uncertainties, and you must consider the effect of these uncertainties on the calculated quantity. For example, the calculated density ρ of a circular cylindrical object is given by

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\pi R^2 L}.$$

But M , R , and L might be measured values with uncertainties ΔM , ΔR , and ΔL . What then is the uncertainty $\Delta\rho$ in ρ ?

For statistical or random errors, a detailed analysis allows you to determine the uncertainty in a calculated quantity in terms of the uncertainties in the measured quantities. If we take a general function f of the variables a , b , c , ..., it can be shown that the uncertainty in f can be calculated by the relationship:

$$(\Delta f)^2 = \left(\frac{\partial f}{\partial a}\right)^2 (\Delta a)^2 + \left(\frac{\partial f}{\partial b}\right)^2 (\Delta b)^2 + \left(\frac{\partial f}{\partial c}\right)^2 (\Delta c)^2 + \dots$$

Again, this relationship assumes all errors are random and are Gaussian in nature. We see that there are two essential contributions from each variable to the uncertainty in f : the partial derivative of f with respect to the variable, and the uncertainty in the variable. Clearly, the greater the uncertainty in a given variable, the greater will be the uncertainty in the derived quantity, f . Also if f depends strongly on a given variable, the uncertainty in that variable will be more important to the uncertainty in f ; hence, the partial derivative appears as a multiplicative factor in each term. Furthermore, note that the uncertainties add in "quadrature" like the components of a vector or the sides of a right triangle.

In case you have never before seen a partial derivative, the concept is really quite simple. The function f depends on several variables; however, in each term of equation (6) we are only interested in one variable at a time. Hence, when we evaluate $\partial f/\partial a$, we treat b , c , ..., as constants while taking the derivative of f with respect to a .

This general formula can be used for any relationship, such as the one for density above. There Hence

$$\begin{aligned}\frac{\partial \rho}{\partial M} &= \frac{1}{\pi R^2 L} = \frac{\rho}{M} \\ \frac{\partial \rho}{\partial L} &= \frac{-M}{\pi R^2 L^2} = -\frac{\rho}{L} \\ \frac{\partial \rho}{\partial R} &= \frac{-2M}{\pi R^3 L} = -\frac{2\rho}{R}\end{aligned}$$

Now let's take a concrete example. You are asked to measure the density of a metal cylinder.

$$\left(\frac{\Delta\rho}{\rho}\right)^2 = \left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + 4\left(\frac{\Delta R}{R}\right)^2.$$

You obtain the following data using a vernier caliper and an electronic scale:

$$M = 185.2 \text{ g}$$

$$L = 6.23 \text{ cm}$$

$$R = 1.05 \text{ cm}$$

Based on the precision of the measuring instruments, you deduce that:

$$\Delta M = 0.1 \text{ g}$$

$$\Delta L = 0.01 \text{ cm}$$

$$\Delta R = 0.01 \text{ cm}$$

Solving for ρ , we have $\rho = 8.58 \text{ g/cm}^3$. Then we may use equation (7) to solve for $\Delta\rho$:

$$\begin{aligned}\Delta\rho &= \rho \sqrt{\left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + 4\left(\frac{\Delta R}{R}\right)^2} \\ &= 8.58 \text{ g/cm}^3 \sqrt{\left(\frac{0.1}{185.2}\right)^2 + \left(\frac{0.01}{6.23}\right)^2 + 4\left(\frac{0.01}{1.05}\right)^2} \\ &= 0.16 \text{ g/cm}^3\end{aligned}$$

Thus we would say that the density of the cylinder is $\rho = (8.58 \pm 0.16) \text{ g/cm}^3$. (By looking at the above expressions, which quantity would you like to measure with greater accuracy? Why?)

Although equation (6) is a general formula that can be applied to any equation, the following expressions are often useful:

1. If $f = \pm a \pm b \pm c$

$$(\Delta f)^2 = (\Delta a)^2 + (\Delta b)^2 + (\Delta c)^2$$

2. If $f = abc$ or $f = a/bc$, etc.

$$\left(\frac{\partial f}{f}\right)^2 = \left(\frac{\partial a}{a}\right)^2 + \left(\frac{\partial b}{b}\right)^2 + \left(\frac{\partial c}{c}\right)^2$$

The quantity $\Delta f/f$ is termed the fractional or relative error in f .

IV. FITTING EMPIRICAL EQUATIONS TO EXPERIMENTAL DATA

If you can find a mathematical equation to describe (very accurately or only approximately) an experimental set of data, you can obtain not only greater insight into the data but also greater utility of the data. For example, you can algebraically manipulate the data and can do computer calculations with much greater ease. The process of finding an empirical equation is simply to make an intelligent guess of the form of the equation. This form will have two or more undetermined parameters that can be adjusted to obtain a desired or best fit to the data such that the differences between the experimental values and those given by the equation are some type of minima.

The three most commonly encountered general forms of equations are given below with some special cases as illustrations:

1. The polynomial expansion, $y = a + bx + cx^2 + dx^3 + ex^4 + \dots$, where the parameters a, b, c, d, e, \dots are to be determined. This form includes the linear equation ($y = a + bx$), the quadratic equation ($y = a + bx + cx^2$), and the simple power equation ($y = px^q$, when q is an integer) as special cases.
2. The simple power equation (also called the Geometric equation) with q not an integer, $y = px^q$, where p and q are the parameters to be determined.
3. The exponential function, $y = he^{kx}$, where h and k are the parameters to be determined and e is the base of the natural logarithms.

Other more elaborate equations are used in special situations.

Computer programs have been developed to fit very complex as well as simple equations to prescribed data. Here the concept is only introduced. The procedure is generally two-step:

1. Plot the raw data to get a feel for what type of curve might fit and then make an intelligent guess of the form of the equation. Often some theoretical arguments will suggest the form of the equation.
2. Determine the undetermined parameters by using one of the following three techniques:
 - a. If there are n undetermined parameters, write the proposed equation n times using selected values of x and y taken from points of a smooth (hand-drawn) curve through the data. These equations will contain the n undetermined parameters and will represent a set of n equations, which can generally be solved for the undetermined parameters. It is always wise to select points (of x and y) that are widely separated on the data curve. Plotting the empirical equation on the data graph for comparison can make a check of the validity of the fit.

- b. Manipulate each data point algebraically to produce a linear curve from which you can extract parameters. Here is an example of this process:

$$y = a + bx^m$$

If you anticipate the data would fit an equation of the form where m is any real number (not necessarily an integer), you should recognize that a is the y -intercept of the $y(x)$ data that have been previously plotted and that a can

$$\log(y - a) = \log(b) + m \log(x)$$

usually be obtained with reasonable accuracy from a smooth hand-drawn curve through the data points extended to the y -axis. Then note that

Thus, if for each data point the values $\log(y-a)$ and $\log x$ are determined, a plot of this modified data with variables $y = \log(y-a)$ and $u = \log x$ should be a straight line ($v = \log b + mu$) with a v -intercept of $\log b$ and a slope of m .

This example is rather complex, but it illustrates that it requires some creativity mixed with logic to know what type of manipulations you need to make to the data before plotting a modified curve. Often you would manipulate the data to obtain a straight line. If you desire to demonstrate that a particular form is appropriate, the approach illustrated here has the advantage that the final curve gives you a conceptual feel for validity of the fit.

- c. A third technique known as a *least-square fit* involves calculating the deviation of each data point from the proposed mathematical curve and minimizing the sum of the squares of these deviations by changing the undetermined parameters in the equation. This procedure involves computer programming but gives the best fit and is the procedure generally used in any research work. Programs using this approach are provided on the computers found in the laboratory, and most of your work will use this technique.