SOUND AND FOURIER ANALYSIS

Apparatus:
Good quality microphones
BNC to microphone connectors
Various sources of sound

Introduction

In Physics 150 we studied a number of ways to analyze experimental data. Such methods included curve fitting and finding standard deviations of data sets. In this week's lab, we will learn about a very powerful method of analyzing oscillatory signals, such as sound waves. This method is called "Fourier Analysis." Although our application will be to sound waves, there are many other applications to Fourier Analysis that you will learn as you continue your study of physics.

Since you are probably unacquainted with Fourier Analysis, we will begin by explaining what it is. As you have learned, we can use a Taylor series to expand any function in terms of a power series. For example, the hyperbolic tangent can be represented as:

$$\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \ldots$$

Similarly, we can express functions in terms of other variables, such as sin(nx) and cos(nx):

$$\tanh(x) = 1.779366 - 0.101041\sin(x) - 0.164288\cos(x) - 0.115860\sin(2x) - 0.085819\cos(2x) - 0.097119\sin(3x) - 0.0428174\cos(3x) - \ldots$$

Any set of functions that can be used for the expansion of an arbitrary function is called a "basis set." For a general function of t, we may write:

$$f(t) = A_0 + A_1\cos(\omega_0 t) + B_1\sin(\omega_0 t) + A_2\cos(2\omega_0 t) + B_2\sin(2\omega_0 t) + A_3\cos(3\omega_0 t) + B_3\sin(3\omega_0 t) + \ldots$$

Here ω_0 is an arbitrary angular frequency used in the analysis. In terms of waves, this tells us that any wave can be made out of a linear superposition of sine and cosine waves. Notice that if the function looks much like sin($2\omega_0 t$), then B_2 will be large; if it looks very little like sin($2\omega_0 t$), then B_2 will be small. Thus the coefficients tell us how much each sine or cosine wave contributes to the overall wave, or equivalently, how much of the overall wave has a given angular frequency, $n\omega_0$. We refer to this process as transforming the wave from the "time domain" to the "frequency domain."

A simple Fourier analysis does nothing more than find the coefficients of sin($n\omega_0 t$) and cos($n\omega_0 t$) in expansions such as the one above. Notice that in this transformation, the only allowed frequencies are ω_0, $2\omega_0$, $3\omega_0$, etc. It is also possible to consider a different version of the Fourier transformation that will allow us to break down the wave into a continuous range of angular frequencies. If you are interested in the math, we use the expansion above and (1) replace a sum over discrete angular frequencies with an integral over continuous angular frequencies, and (2) combine the sine and cosine functions into a complex exponential. The equations are:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega, \quad F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt.$$

Thus $F(\omega)$ takes the place of the A and B coefficients in the expansion above. Also note that since $F(\omega)$ is a complex function, we generally plot the magnitude of F on the screen.

For the purposes of this lab you do not need to understand the details of how a Fourier transformation is performed. You can treat the Fourier transform VIs as “black boxes” - they are complex machines that take an input signal and provide you with an array of the amplitudes of the different frequency components of that signal. Because the input signal is a sampled signal (*i.e.*, we only have values at discrete times) the output array will not be a continuous function of frequency but will be the amplitudes at discrete frequencies that are related to the time interval between the samples in the input data.

In this exercise it is important to remember what you learned in the last experiment about the importance of choosing a correct sample rate for your data. If you do not remember the concept of a Nyquist minimum sampling frequency you should probably go back and review some of the materials from Experiment #2.

Objective: To learn how to use Fourier transforms to analyze the frequency spectra of sounds.

Procedure:

A. LabVIEW – analysis

LabVIEW has a fairly extensive array of tools available for the analysis and manipulation of data once it has been acquired. In previous labs you have studied how to acquire the data. In this lab we will look at some of the available analysis tools. Chapter 10 in the book “Learning with LabVIEW” (found attached to the computer table) covers many of these tools. You should work through sections 10.2 (Curve Fitting), 10.8 (Signal Generation), and 10.9 (Signal Processing - Section 10.9.3 on Filtering is optional). In these sections any reference to a directory such as “Examples\Analysis\regressn.llb” will be found on the hard disk by prefixing “C:\Program Files\National Instruments\Labview\” to it. You would also replace the “LabVIEW” with “C:\Program Files\National Instruments\Labview” in any directory that begins with “LabVIEW.” When they refer to the “Learning” directory you should look in “C:\Phys150-250\Learning” for those files. The “LabVIEW Quick Reference Guide and Getting Started with LabVIEW” books should be available in the lab for your reference.

B. Fourier Analysis of Mathematical Functions

Open the program FOURF by clicking on the icon. With this program, you will create and analyze waves. You may find it helpful to use the magnifying glass to zoom in on the data. After doing this, however, you may not be able to restore the x-axis to its original scale. If this is a problem, you may adjust the scale by clicking on the first number and setting it to 0, and on the last number and setting it to 12. If you have problems, you may want to just exit and reopen the program.

1. Leave the settings for Waves #2 and #3 on zero. Adjust settings for Wave #1. Click on the white "start" arrow to the upper left of the screen. Note that the wave and its Fourier transform both appear in graphs. Explain how each knob affects the wave and its transform.

2. You may adjust all the knobs to create the sum of three waves. Explain what the waves and Fourier transforms are doing as you change the various parameters.

C. Fourier Analysis of Signals

In order to analyze a signal you will need to build a LabVIEW VI which will acquire a signal from a microphone, calculate the Fourier transform, and graph the results. The front panel really doesn't need to have anything on it but the graphs (one for the input signal and one for the Fourier analysis of that signal). The acquisition of a signal will involve a single VI: a single channel, multipoint acquisition. This VI can be found in "Data Acquisition" -> "Analog Input" -> "AI Acquire Waveform.vi" (be careful that you don't get the one for multiple waveforms). You will need to set the "hi limit" and "lo limit" values to something that is appropriate to a microphone signal (about .01 and -.01 respectively - these may need to be adjusted if you find that your signals are clipped). The sample rate must be chosen according to the maximum frequency you will want to observe - the sample rate must be twice the maximum frequency you wish to see. For example, if I wanted to measure frequencies up to 5 kHz, I would need to sample at least 10,000 samples/second. You should take enough samples to cover several seconds of signal. The device number should be specified as 1. The channel number specifies the analog input number you wish to use for your input signal. Remember that you must specify channel "0" if you wish to use analog input number 1 on the interface boxes.

NOTE: You may use channels defined in the DAQ Channel Wizard so that they can be referred to by name rather than device and input number. If you do use those channels be sure that you define the channel to have limits of -.01 to .01 volts so that you have adequate resolution on the signal from the microphone.

The Fourier analysis VI that is most reasonable for this application actually calculates the power spectrum rather than just the Fourier transform. The power spectrum is the square of the magnitude of the Fourier transform so you don't have to worry about complex values that normally result from a Fourier analysis. This VI is found in "Signal Processing" -> "Spectral Analysis" -> "Power Spectrum.vi". This VI takes as an input an array of evenly spaced values or a waveform (such as the waveform that comes from the Input Waveform VI) and gives as an output an array of evenly spaced values where the spacing between the values is the sampling rate for the input signal divided by the number of points in the input array. For instance, if the input array was acquired at a rate of 1000 samples/second (a time spacing of 0.001 seconds), and 1000 points were taken the spacing of the values on the output will be 1 Hz. Remember that the input to this VI is a function of time and the output is a function of frequency.

A feature of the Fourier transform that can cause some confusion is that the output includes the amplitude for both "positive frequencies" (in the first half of the array) and "negative frequencies" (in the second half of the array). This usually results in a graph that appears to be mirrored about the center. The left half represents the "positive frequencies" starting with DC and going up to one half of the sampling frequency. The right half of the graph represents the "negative frequencies" with the frequency nearest zero being at the far right end of the graph. For the example in the previous paragraph (1000 samples/second with 1000 points) the Fourier transform will contain 1000 points. The first 501 points will correspond to the frequencies 0 to 500 Hz and the next 499 points will correspond to the frequencies -499 to -1 Hz. For our purposes we need consider only the "positive frequencies."

If you would like to read more about how the Fourier transform works you may want to look at the documentation on the Fourier Transform in the online help available with LabVIEW.

In order to have graphs with "nice" x-axis values you will need to create a "cluster" of information to pass to the graph as described in chapter 7 of "Learning with LabVIEW." To review, a "cluster" is a collection of more than one type of data (similar to a "structure" in most programming languages). You can create a bundle using the "Bundle" function ("Cluster" -> "Bundle") where the contents of the bundle are described in the online help for the graph (put the cursor over the graph terminal on the wiring diagram and press "control-H"). You should expand the bundle to have 3 elements with the top element being the initial value ("0.0"), the second element being the increment between x values ("1/Sample Rate" for a graph of the data and "Sampling Rate/Total Number of Points" for a graph of the Fourier Transform) and the third element being the array of y values. If you have a "waveform" type of data such as that produced by the "AI Acquire Waveform" VI you do not need to set the x-axis values since that information is contained within the waveform data type and the graph will automatically receive the necessary information to fix the axes. *You should also change the values on both axes of both graphs (input waveform and Fourier transform) to be shown in scientific notation with at least 3 digits of precision (you do this by placing the cursor on one of the numbers on the axis, right-clicking, and selecting "Formatting").*

Once you have created your VI you should look at several sounds:

1. Plug the microphone into the Analog Input on the interface box that you specified when you created your VI. Be sure the interface box is turned on.

2. Make a sound and then click on the start arrow. Try to keep the sound somewhat constant for the duration of the sample period.

3. Describe the information contained in the wave and its transform.

If you have time try several different sounds. When you are looking at speech sounds the "cleanest" are generally sung vowel sounds. You can try different vowels or different people with the same vowel. Remember that the Fourier analysis will only be meaningful if the same sound is held throughout the entire sampling time (as nearly as possible).