Lab 11

Mechanical and Acoustical Resonators

In this lab, you will see how the concept of AC impedance can be applied to sinusoidally-driven mechanical and acoustical systems.

11.1 Mechanical Oscillator

Consider the mechanical system shown in Fig. 11.1. Some sort of external force \( F_{\text{external}} \) pushes on a mass \( m \), which is attached to a spring. Let \( x \) be the amount that the spring is stretched from its equilibrium position. The spring imposes a restoring force, which by Hooke’s law is \( F_{\text{spring}} = -kx \), where \( k \) is called the spring constant. There may also be friction from, for example, air resistance, or perhaps the mass is submerged in a bucket of oil. We assume that the friction is proportional to the velocity of the mass \( v \), but in an opposing direction such that \( F_{\text{friction}} = -bv \).

This system is called a damped mass-spring oscillator. It is subject to Newton’s third law: \( F = ma \). The equation of motion is therefore

\[
F = F_{\text{external}} + F_{\text{spring}} + F_{\text{friction}} = ma \quad \Rightarrow \quad F_{\text{external}} = bv + kx + m \frac{dv}{dt} \tag{11.1}
\]

where we have used the fact that \( a = \frac{dv}{dt} \). This mechanical system is highly relevant to many engineering applications, including acoustics or gently bouncing a baby on a mattress.

The good news is that you already know all about this mechanical system. That is, (11.1) looks just like the equation for a driven series LRC circuit! If we recognize that \( x = \int v dt \), then (11.1) can be written as

\[
F_{\text{external}} = bv + k \int v dt + m \frac{dv}{dt} \tag{11.2}
\]

An analogy with (6.6), \( V = RI + \frac{1}{C} \int I dt + \frac{L}{dt} \frac{dI}{dt} \), is now quite clear. The force \( F_{\text{external}} \) acts like a voltage, while the velocity \( v \) plays the role of current. In this
11.2 Helmholtz Resonator

In all of the impedance-related concepts and equations for the series LRC circuit may be used. We simply make the substitutions $L \rightarrow m, \frac{1}{C} \rightarrow k,$ and $R \rightarrow b.$

The magnetic field surrounding an inductor is hard to get going and hard to stop, so it acts like inertia (mass). Meanwhile, the electrical field in a capacitor always pushes back like the spring. A resistor dissipates energy just like friction.

If a sinusoidal force $F_{\text{external}} = F_0 e^{i \omega t}$ is applied, a comparison with (6.26) and (6.28) reveals

$$v = \frac{F_0 e^{i \omega t}}{b + \frac{k}{i \omega} + i \omega m} = \frac{F_0 e^{i \omega t}}{\sqrt{b^2 + (\omega m - k/\omega)^2}} e^{-i \phi} \quad (11.3)$$

where, similar to (7.4) or (7.10), $\phi = \tan^{-1} \left( \frac{\omega m - k/\omega}{b} \right)$.

The concept of mechanical resonance is immediately apparent. The driving frequency can be chosen to decrease the denominator of (11.3) and thereby maximize the amplitude of oscillations; a well chosen frequency makes a baby on a mattress to go higher.

### 11.2 Helmholtz Resonator

An acoustical resonator, often called a Helmholtz resonator, behaves very similar to a mechanical mass/spring oscillator. It consists of a rigid hollow sphere with a relatively small tube-like opening to the outside. (See Fig. 11.3.) In this case, the compressible gas inside the cavity acts as a spring, while the air within the neck (a tube-shaped opening) acts as the mass. Viscous friction between the air and the walls of the neck introduces damping. The Helmholtz resonator is an especially simple example of an acoustical cavity complete with acoustical resonance. A speaker driver or other sound source just outside the opening excites the resonance inside the cavity.

The resonance frequency of Helmholtz resonator is approximated by

$$f_0 = \frac{c}{2\pi} \sqrt{\frac{A}{V \ell}} \quad (11.4)$$

where $A$ and $\ell$ are the respective cross-sectional area and effective length of the tube-shaped opening, $V$ is the internal volume of the resonator cavity, and $c$ is the speed of sound in air (343 m/s). The effective length of the opening is somewhat longer than the actual length due to the fact that some of the air just inside and just outside the opening is also moving. A good approximation for the effective length is

$$\ell = L + 1.45r \quad (11.5)$$

where $L$ and $r$ are the actual length and radius of the neck. Further discussion as well as a derivation is available at [http://www.phys.unsw.edu.au/jw/Helmholtz.html](http://www.phys.unsw.edu.au/jw/Helmholtz.html).

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1 The energy required to establish a magnetic field in an inductor is $\frac{1}{2} L I^2$, which looks a lot like kinetic energy of a moving mass $\frac{1}{2} m v^2$. The energy required to build up charge in a capacitor is $\frac{1}{2} C Q^2$, which looks a lot like potential energy in a spring $\frac{1}{2} k x^2$. 

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11.3 Equipment

Magnetically-driven mass-spring oscillator system with VI control. Helmholtz oscillator (small speaker, Wavetek signal generator Christmas bulb, lump of clay, computer microphone). Digital frequency meter.
Quiz

Q11.1 In the analogy between an LRC circuit and a mass-spring oscillator, a stiff spring is like a
(a) large inductance.
(b) small inductance.
(c) large capacitance.
(d) small capacitance.

Q11.2 The resonance frequency of the mass/spring oscillator does not depend on
(a) the mass.
(b) the spring constant.
(c) the damping coefficient $b$.

Q11.3 In the analogy between an LRC circuit and an acoustical resonator, a large resonator volume is like a
(a) large inductance.
(b) small inductance.
(c) large capacitance.
(d) small capacitance.

Q11.4 Consider a damped mass-spring oscillator with $m = 50$ g, $k = 80$ N/m, and $b = 25$ g/s that is driven at $f = 2.5$ Hz by a vibrator with peak force amplitude $F_0 = 1$ N.
(a) What is the resonance frequency $f_0$?

(b) Determine the maximum velocity $v_0$ of the mass.

(c) Use energy conservation, $\frac{1}{2}mv_0^2 = \frac{1}{2}kx_0^2$, to determine the maximum displacement $x_0$. 

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Exercises

Half of the class should start on part A, while the other half starts on part B. Roughly halfway through the lab period, the A and B lab teams will switch places.

A: Measure the frequency response of a damped mass-spring oscillator.

L11.1 Working with a team of up to three students, familiarize yourself with the damped mass-spring resonator apparatus. Use either of two available stations. Analog output channel 1 on the interface box drives a current through the large electromagnet, which creates the magnetic force that drives the mass-spring system. Analog input channel 2 (called ai1 by Labview) reads in the voltage from the smaller coil, which is proportional to the velocity of the mass.

L11.2 Given the mass, the spring constant, and the damping coefficient of your apparatus, predict the resonance frequency of your oscillator. The combined mass \( m \) (of the rod, magnet, and damper) is printed on the front of the apparatus. The spring constant \( k \) can be approximated by resting a known mass (between 100 g and 500 g) on the damper and observing the resulting displacement. The approximate damping coefficient \( b \) should be printed on the underside of the damper disc. Enter the resulting \( b, m, \) and \( k \) values into your lab notebook. Use “impedance.nb” to plot the expected ”response” curve vs frequency.

L11.3 Use the “HarmonicOscillator” VI to map out the frequency-dependent velocity of the oscillator over a reasonable frequency range that includes both the low and high-frequency tails of the resonance. Collect about 30 data points, manually recording the peak-to-peak signal amplitude (with estimated error) at each point. Choose an an appropriate linear frequency range that includes the interesting features and which approximately centers the resonance in the middle of range. Enter the data into Excel and plot it for your lab notebook. Show how the measured resonance frequency compares to your predicted value? They should be quite close.

L11.4 Use Mathematica (“NLSQ.nb”) to fit a resonance curve to your data. In contrast to the approach taken with the electrical LRC circuit, fix the mass at the printed value and refine both \( b \) and \( k \), along with the scale factor. Make sure that your result is reasonable and record it in your lab notebook along with error estimates for the fitting parameters.

B: Measure the frequency response of an acoustical Helmholtz resonator.

L11.5 Choose a Helmholtz resonator cavity from among the objects provided. Excite the resonance by blowing a gentle stream of air across the end of
the opening. With a little practice, you may be able to produce a clean musical note, though you shouldn't expect to outshine Fritz Richard after only one day.

**L11.6** Mount a small acoustic driver (i.e. speaker) a short distance away from the mouth of the resonator and drive it with a sinusoidal waveform from the Wavetek signal generator. Search the frequency range from 100 Hz to 600 Hz to locate the Helmholtz resonance. When you find the resonance, the audible sound will get much louder as speaker approaches the cavity. (Don't be fooled by a higher-frequency resonance of your inexpensive speaker driver.) Once you find the resonance, adjust the distance between the driver and the resonator until you maximize the response. Use rods and clamps to fix the speaker in an optimal position.

**L11.7** Measure the acoustic response by dangling a small microphone into the center of the cavity by its leads, and then plug it in to the microphone jack (pink) on the front of your computer. Create a labview VI that reads in a 0.10 second sample from the microphone at a 20 kHz sample rate, and sends the output into a Graph. Use the Express → Input → Sound Input VI, and create numeric controls to set the sample rate and the sample duration. Place these items inside of a “While” loop, and set it to run continuously. If your microphone is not correctly connected, the VI throws and error message and occasionally hangs the computer (blame Windows 7). In the frequency range below 1 kHz, the graph should show a clean sinusoidal signal coming in from your mic that gets updated several times per second.

**L11.8** Map out the frequency response of the resonator over a reasonable frequency range that includes both the low and high-frequency tails of the resonance. Use a digital frequency meter to accurately measure your signal frequency. Collect about 30 data points, manually recording the peak-to-peak signal amplitude at each point. Include amplitude error estimates. Record your data in an Excel spreadsheet and plot the results on a linear horizontal scale for your lab notebook. Note that you don't need to attempt a quantitative fit to the data, though you can if you want to.

**L11.9** Use the expression in the introduction to predict the resonance frequency. How does your prediction compare to the observed resonance frequency? If there is a discrepancy, can you explain it?