Physics 452: Homework #5

Due Tuesday, Jan. 27, 5:00PM, 2009

6.29 HINT: Note that

$$H' = V' = \begin{cases} -\frac{e^2}{4\pi b} + \frac{e^2}{4\pi r} & (r < b) \\ 0 & (r > b) \end{cases}$$

Some useful integrals: $\int xe^{\alpha x}dx = \frac{e^{\alpha x}}{\alpha}\left(x - \frac{1}{\alpha}\right)$, $\int x^2 e^{\alpha x}dx = \frac{e^{\alpha x}}{\alpha}\left(x^2 - \frac{2x}{\alpha} + \frac{2}{\alpha^2}\right)$.

You should obtain $\Delta E = \frac{e^2}{4\pi\varepsilon_o a}\left[\left(1 - \frac{a}{b}\right) + \left(1 + \frac{a}{b}\right)e^{-2b/a}\right]$. Expand the exponent as

$e^{-2b/a} = 1 - \frac{2b}{a} + \frac{2b^2}{a^2} - \frac{4b^3}{3a^3} + \ldots$ and simplify, keeping the lowest-order of b/a that survives.

6.7 HINT: For part (c), simplify your wave functions to a sine or a cosine. When checking the results, the following integrals will be useful: $\int_{-\infty}^{\infty} \sin^2 \beta x e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\left(1 - e^{-\beta^2/\alpha}\right)$, $\int_{-\infty}^{\infty} \cos^2 \beta x e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\left(1 + e^{-\beta^2/\alpha}\right)$.

On part (d), try the parity operator defined by $\hat{P}f(x) \equiv f(-x)$, where the eigen functions are odd and even functions with eigen values ± 1.

6.9 (worth double)