Physics 452: Homework #22

Due Monday, Apr. 13, **Next Morning**, 2009

1. Show that the four-by-four matrices written in short-hand notation $\alpha_x = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}$, $\alpha_y = \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}$, $\alpha_z = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}$, and $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ satisfy the following conditions:

$\alpha_i \alpha_j + \alpha_j \alpha_i = 2\delta_{ij} I$

$\beta \alpha_i + \alpha_i \beta = 0$

$\beta^2 = I$

2. Show that the Dirac can be written as
$\left[\gamma_\mu \frac{\partial}{\partial x_\mu} + k_c I\right]\Psi = 0$, where $k_c \equiv 2\pi/\lambda_c \equiv mc/\hbar$, $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$, $x_4 \equiv -ct$, and the repeated Greek index implies a summation. Determine each of the γ_μ matrices.

3. Show $\vec{\sigma} \cdot \left(-i\hbar\nabla - q\vec{A}\right) \vec{\sigma} \cdot \left(-i\hbar\nabla - q\vec{A}\right) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left(-i\hbar\nabla - q\vec{A}\right)^2 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + q\hbar\vec{\sigma} \cdot \left(\nabla \times \vec{A}\right) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$.

6.19 from the textbook