Physics 452: Homework #20

Due Thursday, Apr. 2, 5:00PM, 2009

1. If $\vec{A}(\vec{r},t) = \hat{z}A_o \sin(kx - \omega t)$ and $\phi(\vec{r}) = \dfrac{e}{4\pi\varepsilon_o r}$, compute $\vec{E}$ and $\vec{B}$.

2. By installing $\vec{A}(\vec{r},t) = A_x(\vec{r},t)\hat{x} + A_y(\vec{r},t)\hat{y} + A_z(\vec{r},t)\hat{z}$ show explicitly that

$\vec{F}_{Lorentz} = q\left[-\nabla\phi - \dfrac{\partial\vec{A}}{\partial t} + \vec{v}\times\left(\nabla\times\vec{A}\right)\right]$ and

$\vec{F}_{Lorentz} = \nabla\left(-mc^2\sqrt{1-v^2/c^2} - q\phi + q\vec{v}\cdot\vec{A}\right) - \dfrac{d}{dt}\nabla_{\vec{v}}\left(-q\phi + q\vec{v}\cdot\vec{A}\right)$ are equivalent.

HINT: Straight away, you can write $\nabla\left(\sqrt{1-v^2/c^2}\right) = 0$, $\nabla_{\vec{v}}(\phi) = 0$, and $\nabla_{\vec{v}}\left(\vec{v}\cdot\vec{A}\right) = \vec{A}$.

Note also that $\dfrac{\partial\vec{A}}{\partial t} \neq \dfrac{d\vec{A}}{dt}$.

3. Nonrelativisitically, we have $\vec{F} = \dfrac{d\vec{p}}{dt} = \dfrac{dm\vec{v}}{dt} = \dfrac{d}{dt}\nabla_{\vec{v}}\left(\dfrac{1}{2}mv^2\right)$. In this case, determine the Lagrangian, canonical momentum, and Hamiltonian for a charged particle experiencing electromagnetic potentials ϕ and $\vec{A}$. Show that quantization leads to the following form of the Schrödinger equation: $i\hbar\dfrac{\partial\Psi}{\partial t} = \dfrac{1}{2m}\left(-i\hbar\nabla - q\vec{A}\right)^2\Psi + q\phi\Psi$.

4. For the fields in problem 1, justify our use of

$\Rightarrow i\hbar\dfrac{\partial\Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + q\dfrac{e}{4\pi\varepsilon_o r}\Psi + q\omega A_o z\cos(\omega t)\Psi$ in chapter 9 of the textbook, starting from

$i\hbar\dfrac{\partial\Psi}{\partial t} = \dfrac{1}{2m}\left(-i\hbar\nabla - q\vec{A}\right)^2\Psi + q\phi\Psi$.

HINT: Note: $\left(-i\hbar\nabla - q\vec{A}\right)^2\Psi = \left(-\hbar^2\nabla^2 + i\hbar q\nabla\cdot\vec{A} + i\hbar q\vec{A}\cdot\nabla + q^2A^2\right)\Psi$ where

$\nabla\cdot\vec{A}\Psi = \left(\nabla\cdot\vec{A}\right)\Psi + \vec{A}\cdot\nabla\Psi$. Also, $\nabla\cdot\vec{A} = 0$ for our field. Next let $\Psi(\vec{r},t) \equiv \tilde{\Psi}(\vec{r},t)e^{-i\frac{q}{\hbar}\vec{A}\cdot\vec{r}}$, and obtain an equation for $\tilde{\Psi}$. You may also make the dipole approximation $kx \ll 1$.