

Physics 452: Homework #16

Due Thursday, Mar. 12, 5:00PM, 2009

9.12 HINT: In deriving $[L^2, z] = 2i\hbar(xL_y - yL_x - i\hbar z)$, the following trick is handy:

$[L_x^2, z] = L_x[L_x, z] + [L_x, z]L_x$, and similar for $[L_y^2, z]$ and $[L_z^2, z]$. Keep in mind

$\bar{L} = \hat{x}(yp_z - p_yz) - \hat{y}(xp_z - p_xz) + \hat{z}(xp_y - p_xz)$ for working out $[L_x, z]$, $[L_y, z]$, and $[L_z, z]$ (see also Eq. 2.51).

For the second half of the problem, you will also need $[L^2, x] = 2i\hbar(yL_z - zL_y - i\hbar x)$ and

$[L^2, y] = 2i\hbar(zL_x - xL_z - i\hbar y)$, which are cyclic permutations of what you just derived. Don't bother re-deriving these; just use them.

Now, going forward:

$$\begin{aligned} [L^2, [L^2, z]] &= 2i\hbar([L^2, xL_y] - [L^2, yL_x] - i\hbar[L^2, z]) \\ &= 2i\hbar([L^2, x]L_y + x[L^2, L_y] - [L^2, y]L_x - y[L^2, L_x] - i\hbar(L^2z - zL^2)) \end{aligned}$$

Two of the terms are zero, and two have pieces you can replace with the formulas mentioned above. Afterwards, use also the following replacement: $L_x^2 + L_y^2 = L^2 - L_z^2$.

You should arrive at

$$[L^2, [L^2, z]] = 2\hbar^2(zL^2 + L^2z) - 4\hbar^2\{yL_zL_y - i\hbar xL_y + xL_zL_x + i\hbar yL_x + zL_z^2\}$$

To get the desired answer, we hope that the stuff in the curly brackets is zero. Use Eq. 9.68 in place of $-i\hbar x$ and $i\hbar y$, and things simplify quickly to $L_z \bar{r} \cdot \bar{L}$, whereupon the hint in the book gets you the rest of the way.

9.13

9.14 (Worth double) HINT: You will find that the three possible channels have equal transitions rates; please show this.