Physics 451: Homework #8

Due Tuesday, Oct 7, 5:00PM, 2008

2.23; 2.26 (together worth one – both easy)

2.27 From Prob. 2.1 (c), we know that an even potential must have even or odd solutions, so we may write

$$\psi(x)=\begin{cases} Ae^{-\kappa x} & (x>a) \\ B\left[e^{\kappa x}\pm e^{-\kappa x}\right] & (-a<x<a) \\ \pm Ae^{-\kappa x} & (x<-a) \end{cases}$$

, where the upper sign corresponds to an even solution and the lower sign corresponds to an odd solution. It is enough to pin down the coefficients at the point $x=a$, and we indirectly pin down what goes on at $x=-a$. By insisting that $\psi(x)$ be continuous at $x=a$, we get $Ae^{-\kappa a}=B\left[e^{\kappa a}\pm e^{-\kappa a}\right]$. By integrating the T.I.S.E. from $a-\varepsilon$ to $a+\varepsilon$, we find $-\frac{\hbar^2}{2m}\left[-\kappa Ae^{-\kappa a}-\kappa B\left(e^{\kappa x}\pm e^{-\kappa x}\right)\right]-\alpha Ae^{-\kappa a}=0$. Solving these together, we deduce $e^{-2\kappa a}=\pm\left[\frac{\hbar^2\kappa}{m\alpha}-1\right]$. When $\alpha=\frac{\hbar^2}{ma}$, then $\kappa a=\begin{cases}1.11\\0.80\end{cases}$ works. When $\alpha=\frac{\hbar^2}{4ma}$, then $\kappa a=\begin{cases}0.37\\0\end{cases}$ works, but $\kappa a=0$ leads to zero binding energy!

2.29

2.34