2.23; 2.26 (together worth one – both easy)

2.27 From Prob. 2.1 (c), we know that an even potential must have even or odd solutions, so we may write

\[ \psi(x) = \begin{cases} 
A e^{\pm \kappa} & (x > a) \\
B \left[ e^{\kappa x} \pm e^{-\kappa x} \right] & (-a < x < a) \\
\pm A e^{\kappa} & (x < -a) 
\end{cases} \]

where the upper sign corresponds to an even solution and the lower sign corresponds to an odd solution. It is enough to pin down the coefficients at the point \( x = a \), and we indirectly pin down what goes on at \( x = -a \). By insisting that \( \psi(x) \) be continuous at \( x = a \), we get \( A e^{\kappa a} = B \left[ e^{\kappa a} \pm e^{-\kappa a} \right] \). By integrating the T.I.S.E. from \( a - \epsilon \) to \( a + \epsilon \), we find

\[ \frac{\hbar^2}{2m} \left[ -\kappa A e^{\kappa a} - \kappa B \left( e^{\kappa a} \pm e^{-\kappa a} \right) \right] - \alpha A e^{-\kappa a} = 0. \]

Solving these together, we deduce \( e^{-\kappa a} = \pm \left[ \frac{\hbar^2 \kappa}{m \alpha} - 1 \right] \). When \( \alpha = \frac{\hbar^2}{ma} \), then \( \kappa a = \begin{cases} 
1.11 & \text{works. When } \\
0.80 & \end{cases} \)

\( \frac{\hbar^2}{4ma} \), then \( \kappa a = \begin{cases} 
0.37 & \text{works, but } \\
0 & \kappa a = 0 \text{ leads to zero binding energy!} \end{cases} \)

2.29

2.34