1. **(This problem is worth 2.)** Show that, if you do not make approximation 3 on page 456, Eqs. (11.51) and (11.54) lead to (with Q=0)

\[
\vec{E}(\vec{r}, t) \equiv \frac{\mu_o}{4\pi r} \left[ (\hat{\vec{r}} \cdot \ddot{\vec{p}}) \hat{\vec{r}} - \vec{p} \right] + \frac{\mu_o c^2}{4\pi r^3} \left[ 3(\hat{\vec{r}} \cdot \ddot{\vec{p}}) \hat{\vec{r}} - \vec{p} \right] + \frac{\mu_o c}{4\pi r^2} \left[ 3(\hat{\vec{r}} \cdot \ddot{\vec{p}}) \hat{\vec{r}} - \vec{p} \right]
\]

and

\[
\vec{B}(\vec{r}, t) \equiv -\frac{\mu_o}{4\pi rc} \left[ (\hat{\vec{r}} \times \ddot{\vec{p}}) \right] - \frac{\mu_o}{4\pi r^2} \left[ (\hat{\vec{r}} \times \ddot{\vec{p}}) \right].
\]

These describe the fields relatively close to the dipole in addition to the fields in the far-away limit given by Eqs. (11.56) and (11.57).

HINT: First prove formulas \( \nabla \cdot \vec{f}(\vec{r}) = \hat{\vec{r}} \cdot \left( \frac{\partial \vec{f}(\vec{r})}{\partial \vec{r}} - \frac{\vec{f}(\vec{r})}{r} \right) \hat{\vec{r}} + \frac{\vec{f}(\vec{r})}{r} \)

and \( \nabla \times \vec{f}(\vec{r}) = \hat{\vec{r}} \times \frac{\partial \vec{f}(\vec{r})}{\partial \vec{r}} \). Recall that \( \vec{p} \) and its derivatives are evaluated at the retarded time \( t_o = t - r/c \).

**Extra Mile – Not Required** A driving force causes an electric dipole to oscillate sinusoidally (i.e. \( \vec{p}(t) = \hat{\vec{z}} z_0 \omega \sin(\omega t) \)). The fields in the near vicinity of the dipole (found in the previous problem) work against the agent that maintains the oscillations. Show that the average power required of this driving agent \( P_{driving}(t) = F \cdot v = -E_z \frac{\partial p(t)}{\partial t} \) is the same as the average power radiated away. You need the *near-field* terms of the above E-field only; drop the *far-field* term that appears in Eq. (11.56). Your answer should agree with Eq. (11.22), which is calculated exclusively from the far-field term that does not contribute!

HINT: You will need to invoke \( z_o \ll \lambda \) on which the dipole field formula is based. This will allow you to simplify the following combination that you will encounter:

\[
\frac{\sin \alpha}{\alpha} - \cos \alpha \equiv \frac{\alpha - \alpha^3/3!}{\alpha} - \left( 1 - \alpha^2/2! \right) = \alpha^2/3, \text{ where } \alpha \text{ contains } z_o \omega/c. \]

Note that we seem to be violating approximation (11.45) when using the formula this way.