Review 75

a) A monochromatic plane wave with intensity I_0 and wavelength $\lambda$is incident on a circular aperture of diameter l followed by a lens of focal length f. Write the intesity distribution at a distance f behind the lens

The electric field given by fraunhofer diffraction with cylindrical symmetry is:

$$E(\rho, z) = -\frac{2\pi i e^{ikz} e^{\frac{ik\rho^2}{2z}}}{\lambda z} \int_{aperture} \rho' d\rho' E(\rho', 0) J_0\left(\frac{k\rho\rho'}{z}\right)$$

Equation 10.28 in the book. In chapter 11, we found that putting a lens after the aperture will cause a diffraction pattern that would normally appear at z to appear at the focal length f. Thus we can replace all $z's$ with $f's$ in the equation above and carry out the integral with $E(\rho', 0) = E_0$. Therefore:

$$E(\rho, z) = -\frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \int_{aperture} E_0 \rho' d\rho' J_0\left(\frac{k\rho\rho'}{f}\right)$$

$$= -E_0 \frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \int_0^{\frac{l}{2}} \rho' d\rho' J_0\left(\frac{k\rho\rho'}{f}\right)$$

$$= -E_0 \frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \frac{\frac{l}{2}}{\frac{k\rho}{f}} J_1\left(\frac{k\rho l}{2f}\right)$$

by integral 0.58 in the book. Taking the intensity:

$$I(\rho, z) = E_0^2 \frac{4\pi^2}{\lambda^2 f^2} \frac{\frac{l^2}{4}}{\frac{k^2\rho^2}{f^2}} J_1\left(\frac{k\rho l}{2f}\right)^2$$

$$= I_0 \frac{\pi^2 l^4}{\lambda^2 f^2} \left[2\frac{J_1\left(\frac{k\rho l}{2f}\right)}{\frac{k\rho l}{2f}}\right]^2$$

b) You wish to spatially filter the beam such that, when it emerges from the focus, it varies smoothly without diffraction rings or hard edges. A pinhole is placed at the focus, which transmits only the central portion of the Airy pattern (inside the first zero). Calculate the intensity pattern at a distance f after the pinhole...

We are adding another pinhole, so we will take another integral over another aperature:

$$E(\rho, z) = -\frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \int_{aperture} \rho' d\rho' E(\rho', 0) J_0\left(\frac{k\rho\rho'}{f}\right)$$

Where $E(\rho', 0) = E_f e^{-\frac{\rho'^2}{w_0^2}}$. Integrating, we get:

$$E(\rho, z) = -\frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \int_{aperture} \rho' d\rho' E_f e^{-\frac{\rho'^2}{w_0^2}} J_0\left(\frac{k\rho\rho'}{f}\right)$$

$$= -E_f \frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \int_{aperture} \rho' d\rho' e^{-\frac{\rho'^2}{w_0^2}} J_0\left(\frac{k\rho\rho'}{f}\right)$$

$$= -E_f \frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \frac{e^{-\left(\frac{k\rho}{f}\right)^2 w_0}}{2/w_o}$$

By integral 0.59. Note here that $w_0 = \frac{2\lambda f^{\#}}{\pi}$ and $f^{\#} = f/l$, so the answer becomes:

$$= -E_f \frac{2\pi i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{\lambda f} \frac{e^{-\left(\frac{k\rho}{f}\right)^2 \frac{2\lambda f}{l\pi}}}{2l\pi/2\lambda f}$$

$$= -E_f \frac{i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{l} e^{-\left(\frac{k\rho}{f}\right)^2 \frac{4f}{lk}}$$

$$= -E_f \frac{i e^{ikf} e^{\frac{ik\rho^2}{2f}}}{l} e^{-\frac{4k\rho^2}{lf}}$$