1. In a four level system, the population difference is

$$N_2 - N_1 = \frac{(\Gamma_{10}P - \Gamma_{21}P)N_T}{\Gamma_{21}\Gamma_{21} + \Gamma_{10}P + \Gamma_{21}P}$$

a. Show that if $\Gamma_{21} \ll \Gamma_{10}$, the population difference can be written

$$N_2 - N_1 = \frac{P}{\Gamma_{21} + P}N_T$$

b. Use the expression for the threshold gain to write the pumping rate required to reach threshold.

c. Cr:Forsterite is a four-level system with $\sigma = 1.4 \times 10^{-19}$ cm^2 and $\Gamma_{21} = 3.7 \times 10^7$ s^{-1}. Suppose your laser has one perfectly reflecting mirror and one with $t = 0.03$. The Forsterite crystal is 1 cm, and the density of Cr ions in the crystal is $N = 6 \times 10^{18}$ cm^{-3}. What is the minimum pumping rate required to reach threshold?

2. Starting from the equation for output intensity of a laser,

$$I^{\text{out}} = tI_{\text{sat}} \left(\frac{g_0}{g_t} - 1 \right)$$

show that the output power of a three-level laser is linearly proportional to the pumping rate above threshold, and find the slope efficiency. (If you write the output intensity as a function of the pump intensity, the slope efficiency is the coefficient that multiplies the pump intensity. You can guess the mode area to be $\pi w_0^2/2$.)

3. You have a 1 cm diameter laser beam that carries a peak power of 0.1 terawatts when it is pulsed. You wish to focus it to a peak intensity of 1×10^{16} W/cm^2.
(a) What focal length lens should you use, assuming the beam will focus to the diffraction limit?
(b) What will be the Rayleigh range?
(c) Take the focal volume near the highest intensities to be roughly $V = w_o^2 z_o$. How many atoms are in this volume if the laser is focused in 10^{-6} torr of helium (at 300K)?

4. You wish to construct a ring laser cavity using a 2m focal length lens and flat mirrors as shown below. Let L be an entire trip around the cavity including the effective length of the gain medium (so you can ignore it).
(a) What range(s) of L is possible for making a stable cavity?
(b) If the wavelength is 1054nm, and if the gain medium is centered opposite from the lens, what is the necessary distance L that makes the beam radius in the amplifier be 200μm? HINT: You can use the ABCD law at the lens. Also, please choose the more realistic option for L.
(c) Light travels in both directions in the cavity and exits through an output coupler. Which exiting beam will converge to a focus outside of the cavity? What will the beam waist for that focus be assuming the parameters in part (b)? Where will it be located after the output coupler in terms of L and d?