Physics 105
Math Review

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To Stacy Day
April 21, 2007

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April 21, 2007

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Dear Professor Day:

We are writing to submit final project, *Physics 105 Math Review.* This manual is written to people who are going to or are currently enrolled in Physics 105. Students struggle with the math required to succeed in this course. Many struggle because they haven't had a math course in some time. We wrote this manual to help them remember and apply pertinent concepts.

To our knowledge, a manual like ours doesn't exist here at BYU. There are other manuals that discuss physical and mathematical concepts, but they aren't geared directly to Physics 105 students. Currently there is a manual that covers vectors, but students lack a working knowledge of algebra and trigonometry. Without this knowledge, students find the manual confusing. Our manual reviews basic algebra and trigonometry skills which are key concepts that need to be mastered in order to be successful in Physics 105. Following a review of these areas we have included an introduction section to vectors. Our manual will also provide suggestions which will aid students with this course.

After conferencing with current physics professors, we decided that our original idea of a complete Physics 105 manual would not be as useful as an in depth math review. While preparing the actual manual, we realized that time constraints made this a better option. The math that we review in the manual is common knowledge to those who are in physics and engineering. Therefore, we did not find it necessary to reference outside sources. Jarom's experience as a TA for the class allowed him to know what is important to understand for Physics 105. Our combined knowledge was sufficient to provide a thorough review. We agreed in our rough draft conference with you that we would serve as the primary sources of information and no citing of sources would be necessary. All information and figures, excepting some Microsoft clipart, are original. This manual has been tested by current Physics 105 TA's which work with Jarom. Upon completion of the manual, we gave it to Dr. Gee and Dr. Magleby to test for usefulness in their course. They plan to make it available online in PDF format.

Please feel free to contact us with any questions or concerns you may have. You can reach Jarom at (801) 705-6505 or by email at *jaromgiraud@gmail.com.* You can reach Chris at (801) 499-1868 or by email at *chris24welsh@gmail.com*.

Sincerely,

Table of Contents

Letter of Transmittal ii

Preface iv

Helpful Hints v

Algebra 1

Trigonometry 5

Vectors 13

Dimensional Analysis 17

Preface

The *Physics 105 Math Review* manual is intended to teach basic math skills used in Physics 105. These concepts are necessary to be successful in the course. This manual is designed to help any student who is going to or who is currently enrolled in Physics 105 to better understand simple, but essential concepts. These concepts include algebra, trigonometry, vectors, and dimensional analysis. Many students are required to take Physics 105, but there is a lack of materials to help them. This course could be considered an obstacle without the proper preparation.

This manual is intended to be used as a reference manual. It is to be read all the way through once and then used as a reference. It consists of several sections that go over concepts that are needed to be successful in the class. Each section begins with a detailed explanation of the section's main topic. Following the explanations are examples that are worked out in detail to see the concepts in action. Several sample problems finish out nearly every section so the student can put the newly learned skills into action.

How to use this manual

There are sample problems at the end of most sections. After a student has read about the concepts, he/she can do the sample problems to verify that they understand the concepts. Each sample problem has the problem on the left side of the page with an explanation of how to solve the problem in the right column. The student should cover up the solution on the right side while working out the problem in the left column. Once the student has worked out the problem, he/she can compare their work with the solution to get instant feedback. This way the student can tackle the problem by themselves and verify if they got it right.

Helpful Suggestions...

Physics 105 can be a difficult course, especially if you have little to no experience in the subject. Listed below are some helpful tips that, if followed, will help you achieve greater success. These suggestions relate to the class in general, not specifically to this manual.

...To Understand Concepts

- **Keep current in the class**
  The problems you solve in the homework will be directly related to the material your professor is covering. If you take good notes, chances are you will have written down the exact equation you need to solve your problem. Reading the book will also help you to understand homework problems and difficult concepts.
- **Do the homework and labs**
  Doing the homework and labs will help you stay current with the class and help you to internalize and remember what you have learned. If you get behind, the class will seem harder.
- **Use the tutorial lab**
  Available to everyone, free of charge, is the physics tutorial lab. T.A.s are available to help you understand and apply the concepts you are learning. The tutorial lab is also a great place to work with others in your class.

...To Approach a Homework Problem

1. **Read the problem**
   Before solving any problem, read it first. You will find that many times important information is given in the last sentence. Reading the problem aloud is also beneficial.
2. **Write what you know**
   As you read the problem, write down whatever information it gives you. This practice will help prevent you from glossing over something important.
3. **Draw a picture**
   It doesn't matter how something looks, draw a picture and label all the information you know or want to know. This way, you have another view of the problem and you've written out important information which you can easily refer too.
4. **Decide upon a basic concept**
   From what you've written, drawn, heard or read, decide which idea the teacher wants you to work with. **Most Physics 105 problems deal with either force, energy or momentum.** Pick a concept and start to think how it applies to your question/problem.
5. **Use your book**
   The back of the book has an index which can be used to direct you to explanations and equations.

Algebra

Algebra is the method that we use to solve problems with one or more unknown variables using skills such as addition, subtraction, multiplication, division, etc.

Most of the algebra you use in physics 105 can be remembered in one rule: **Do unto one side as you do to another.**

Example: Solve for T (make an equation where T=something)

$$X = V_0 t + \frac{1}{2} aT^2$$

1. Subtract Vo*t from both sides: $X - V_0 \cdot t = \frac{1}{2} aT^2;$

2. Multiply both sides by 2: $2X - 2V_0 t = aT^2$

3. Divide both sides by "a": $\frac{2X - 2V_0 t}{a} = T^2$

4. Take the square root of both sides to see: $T = \sqrt{\frac{2X - 2V_0 t}{a}}$

So long as you do not violate algebra/math rules, the steps to solving a problem can be performed in whichever order you choose. Be aware that some methods of solving are more efficient than others. You will get better at determining the best order with practice.

Plugging in the Numbers

Now, if we are given the information:
X=55, Vo=12.6, t=4.1, a=36, we can find our final value of T.

$$T = \sqrt{\left(\frac{2 \cdot 55 - 2 \cdot 12.6 \cdot 4.1}{36}\right)} = .430$$

REMEMBER:

1. Physics uses a lot of variables, learning to **solve equations in variable form first**, before plugging in numbers, will save you a lot of time, should you make a mistake.
2. Variables and numbers follow the same rules in algebra. A variable is just a number that hasn't been assigned yet. Don't let symbols intimidate you.
3. **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally**:** 1)**P**arenthesis 2)**E**xponents 3)**M**ultiply/**D**ivide 4)**A**dd/**S**ubtract. These are your order of operations for plugging in numbers to find your final answer.

Collecting Terms

Some problems you will encounter can only be solved with this skill, you will find it useful. If two variables are multiplied by a common variable/number, you can combine terms as seen in this example:

$$C = Bx + \frac{x}{A}$$

1. Notice both B and 1/A are both multiplied by x, so we can factor x out to get:

$$C = \left(B + \frac{1}{A}\right)x$$

2. Now we can divide both sides by (B+1/A) to solve for x.

$$\frac{C}{B + \frac{1}{A}} = x$$

Step 1 is what we call collecting terms, combining constant terms (such as B and 1/A) to leave us with just one X in our equation.

Expanding Terms

Expanding terms is the opposite of collecting them. Suppose we want to solve the following equation for X:

A(X+B)=C*X+D

A(X+B) really means that we have A multiplied by both X and B, this can be expanded to show:

A(X+B) = A*X+A*B

Therefore:

A*X+A*B = C*X+D

Now we can collect like terms and solve for X:

Subtract A*X from both sides: A*B = C*X+D-A*X

Subtract D from both sides: A*B–D = C*X-A*X

Collect terms on the right side: A*B–D = (C-A)X

Divide by C-A to solve for X:

$$\frac{A*B-D}{(C-A)} = X$$

Notice that when solving equations with multiple X's (or other variables),, it is important to get all the X's to the same side of the "=" sign.

Systems of Equations

You will find yourself needing to solve equations which contain more than one variable that you do not know. When this happens, you need to find another equation to help you solve the problem. **For every unknown variable you have, you must have an equation.** 2 unknowns requires 2 equations, 3 unknowns requires 3. This process is best shown by example.

Suppose A, B, C, and D are constants, but Z and X are variables. How can you solve this equation for X?

$$A + B = \frac{ZX}{C - \mathrm{D}}$$

Multiply both sides by (C-D) and then divide by Z to get the result:

$$\frac{(C - \mathrm{D})\,(A + B)}{Z} = X$$

This is good but we are still unable to solve the problem because we do not know Z. It is impossible unless we are given another equation:

$$Z - A = B$$

Now, we can solve this second equation for Z:

$$Z = A + B$$

Substitute D/A for Z in our first equation that we already solved for X and find:

$$C - \mathrm{D} = X$$

Hurray, It simplified!

Physics has numerous equations for solving various types of problems. If you think you don't have the equation to solve a problem, try combining a couple.

Sample Problems

1. Solve for X:

$$A = \frac{X}{\frac{X}{B} + C}$$

Solution

Multiply both sides by (X/B +C)

$$A\left(\frac{X}{B} + C\right) = X$$

Expand the left hand side, multiplying by A

$$\frac{A\,X}{B} + A\,C = X$$

Move all your variables (X) to one side by Subtracting (A*X)/B from both sides

$$A\,C = X - \frac{A\,X}{B}$$

Combine terms, pulling out X

$$A\,C = \left(1 - \frac{A}{B}\right)X$$

Finish by dividing both sides by (1-A/B)

$$\frac{A\,C}{1 - \frac{A}{B}} = X$$

1. Solve for X:

Eq. 1 $10 + Z = 30\,X$

Eq. 2 $Z + 25 = X$

Solve Eq.1 for Z by subtracting 10 from both sides of the equation

$$Z = 30\,X - 10$$

Substitute this new equation in for the Z of Eq. 2

$$30\,X + 15 = X$$

Subtract 30X from both sides

$$15 = -29\,X$$

Divide both sides by -29 to solve for X

$$-\frac{15}{29} = X$$

Trigonometry

The Right Triangle

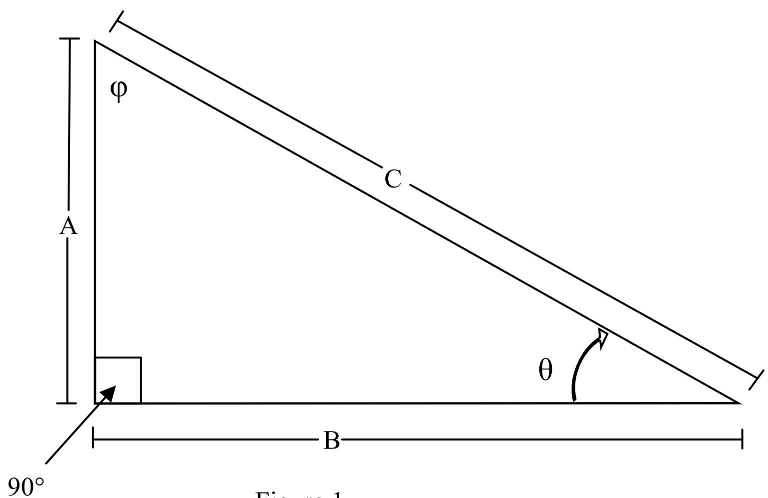


Figure 1

Figure 1 is a basic right triangle. A right triangle consists of a 90° internal angle, represented by the square in the lower left corner of the figure. The square on the interior angle of a triangle will always represent a 90° angle. The side opposite the 90°, side C, is the hypotenuse. Sides A and B are referred to as the legs of the triangle. The other two angles, φ and θ, are the other two interior angles of the triangle. The sum of these two angles is 90°.

Pythagorean Theorem

The three sides of any right triangle can be related to each other by the Pythagorean Theorem. The Pythagorean Theorem can only be used with right triangles. The theorem states that the sum of the squares of the two legs equals the square of the hypotenuse or:

$$A^2 + B^2 = C^2$$

If the length of the two legs are known, the length of the hypotenuse can be found by solving the above equation for C:

$$\sqrt{A^2 + B^2} = \sqrt{C^{\cancel{2}}} = C$$

So

$$C = \sqrt{A^2 + B^2}$$

If the length of the hypotenuse and one of the legs, B, is known then you can solve for the other leg, A:

$$A^2 + B^2 = C^2$$
$$\underline{\qquad - B^2 \quad - B^2}$$
$$A^2 = C^2 - B^2$$

So

$$A = \sqrt{C^2 - B^2}$$

Example

Steve got a great deal on a triangular plot of land (Figure 2). He goes to the hardware store to buy some fencing. Once there he realizes that he only measured the two shorter sides. What is the length of the unmeasured side? How many feet of fencing should Steve buy?

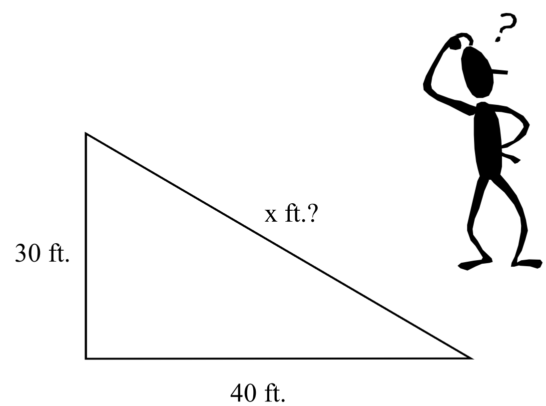


Figure 2

Solution

We are given the value of the two legs of the right triangle that makes up Steve's plot of land. A = 30 ft. and B = 40 ft. He forgot to measure C or the hypotenuse.

From the Pythagorean theorem we know that:

$$C = \sqrt{A^2 + B^2}$$

Plug in the values that we know:

$$C = \sqrt{(30 \text{ ft.})^2 + (40 \text{ ft.})^2}$$

Carry out the operation: $C = \sqrt{900 \text{ ft.}^2 + 1600 \text{ ft.}^2}$

$$C = \sqrt{2500 \text{ ft.}^2}$$

$$C = 50 \text{ ft.}$$

To find out how much fencing Steve needs to buy, simply add up the three sides to find the perimeter:

Perimeter = A + B + C = 30 ft. + 40 ft. + 50 ft.= 120 ft.

Trigonometry

The sides and interior angles of a right triangle can be related to each other through trigonometry. This is useful when you are missing information about a triangle. For example if you know the one interior angle and one side you can find the length of the other two sides and the other interior angle. Also, if you know the length of two sides of the triangle, you can find the two interior angles and the length of the other side.

The angles and sides of a right triangle can be related through the sine, cosine and tangent functions. The two shorter sides are related to the angles as either adjacent or opposite (Figure 3). If the side touches the angle, it is adjacent. If the side doesn't touch the angle it is called opposite. The longest side, opposite to the 90° angle is always called the hypotenuse. The sine, cosine and tangent functions are expressed as follows:

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \tan(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Or

$$\sin(\theta) = \frac{\text{Side A}}{\text{Side C}} \qquad \cos(\theta) = \frac{\text{Side B}}{\text{Side C}} \qquad \tan(\theta) = \frac{\text{Side A}}{\text{Side B}}$$

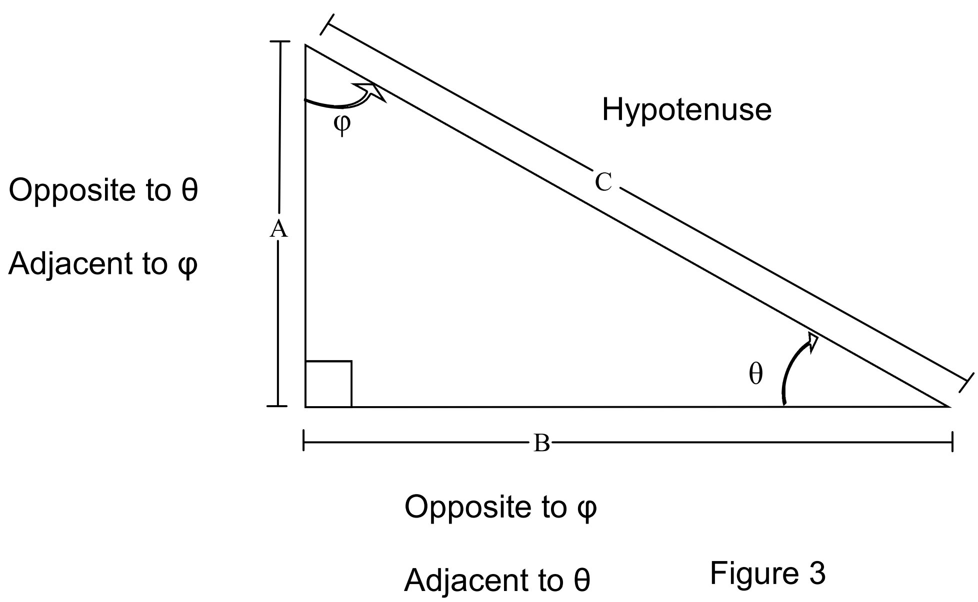


Figure 3

SOH-CAH-TOA

A good way to remember how to use sines and cosines is through a simple saying, SOH-CAH-TOA. This is an easy way to remember when to use which combination of the opposite side, adjacent side, or hypotenuse. The 'S' in SOH stands for sin, the 'O' for opposite and the 'H' for hypotenuse. So when you say SOH, you remember that the sin function is opposite over adjacent or $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$. Similarly, CAH says that the cos function is adjacent hypotenuse. TOA stands for the tan function being opposite over adjacent. If you can memorize this saying, you will always know how to use the sin, cos, and tan functions. Just say it over and over in your head. A good way to remember SOH-CAH-TOA is by also memorizing the sentence, "**O**h **h**eck! **A**nother **h**our **o**f **a**lgebra!"

Sin Cos

Tan

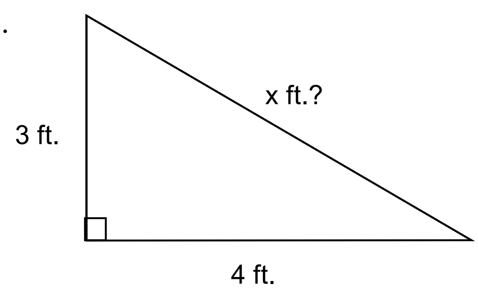
Sample Problems

Solution

Pythagorean Theorem

Find the length of the missing sides in the following problems. Work out the problem, covering up the solution in the right column, and then check your work.

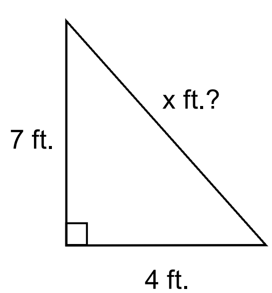
1.



We are given the two shorts sides and asked to find the hypotenuse. Use the relationship that $\sqrt{C = A^2 + B^2}$

$x = \sqrt{4^2 + 3^2}$
$x = \sqrt{16 + 9}$
$x = \sqrt{25}$
$x = 5$ ft. (solution)

2.



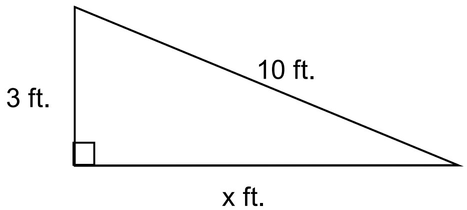
Again we are given the two shorts sides and asked to find the hypotenuse. Use the relationship that $\sqrt{C = A^2 + B^2}$

$x = \sqrt{7^2 + 4^2}$
$x = \sqrt{49 + 16}$
$x = \sqrt{65}$
$x = 8.06$ ft. (solution)

Sample Problems

Solution

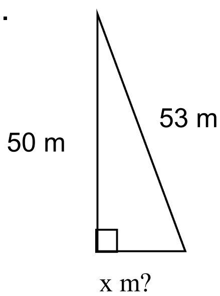
3.



We are given one short side and the hypotenuse. We are asked to find the remaining short side. Use the relationship that
$B = \sqrt{C^2 - A^2}$

$x = \sqrt{10^2 - 3^2}$
$x = \sqrt{100 - 9}$
$x = \sqrt{91}$
x = 9.54 ft. (solution)

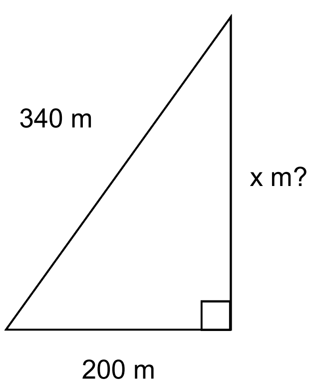
4.



Again we are given one short side and the hypotenuse. We are asked to find the remaining short side. Use the relationship that
$B = \sqrt{C^2 - A^2}$

$x = \sqrt{53^2 - 50^2}$
$x = \sqrt{2809 - 2500}$
$x = \sqrt{309}$
x = 17.58 m (solution)

5.



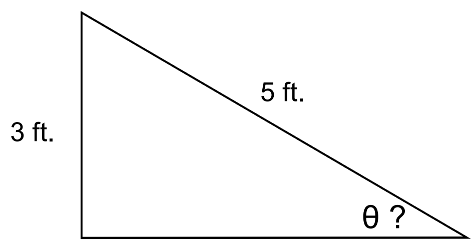
Again we are given one short side and the hypotenuse. We are asked to find the remaining short side. Use the relationship that
$A = \sqrt{C^2 - B^2}$

$x = \sqrt{340^2 - 200^2}$
$x = \sqrt{115{,}600 - 40{,}000}$
$x = \sqrt{75{,}600}$
x = 275 m (solution)

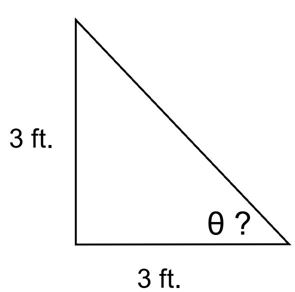
Sample Problems

In the following problems, use the given triangles to find the missing angles or sides. Again work out the problem, covering up the solution in the right column, and then check your work.

6.



7.



Solution

Here we are asked to find the angle θ. They give us two sides. The side that is 3 feet is opposite to the angle. The side that is 5 feet is the hypotenuse. Remember the saying that we memorized to remember the relationships between sides and angles, SOH-CAH-TOA. Since we are given the lengths of the opposite and hypotenuse sides, we will use the sin function (SOH).

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Plug in what you know:

$\sin(\theta) = 3/5$ (You can find the actual angle by using the inverse sin function)

$\sin^{-1}(\sin(\theta)) = \sin^{-1}(3/5)$

$\theta = 36.87°$ (Solution)

Again we are asked to find the angle θ. The give us two sides. One of the given sides is adjacent to the angle and the other is opposite. Remember SOH-CAH-TOA. Since we are given the lengths of the opposite and adjacent sides, we will use the tan function (TOA).

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

Plug in what you know:

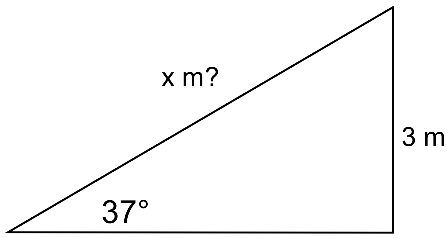
$\tan(\theta) = 3/3$ (You can find the actual angle by using the inverse tan function)

$\tan^{-1}(\tan(\theta)) = \tan^{-1}(1)$

$\theta = 45°$ (Solution)

Sample Problems

8.



Solution

Here we are asked to find the length of the hypotenuse. They give us one of the sides and an interior angle. The side that is 3 meters is opposite to the angle. Remember SOH-CAH-TOA. Since we are given the lengths of the opposite side and we need to find the hypotenuse, we will use the sin function (SOH).

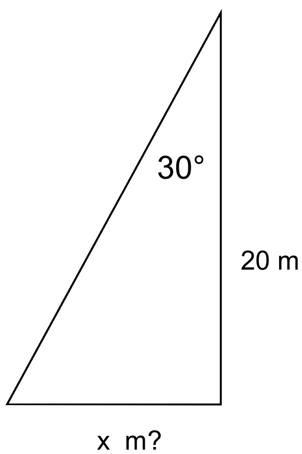
$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Plug in what you know:

$\sin(37°) = 3/x$ (Use algebra to solve for x)

$x = 3 / \sin(37°)$
$x = 4.98$ m (Solution)

9.



Here we are asked to find the length of one of the shorter sides. They give us the other shorter side and an interior angle. The side that is 20 meters is adjacent to the angle and the side in question is opposite to the angle. Remember SOH-CAH-TOA. Since we are given the lengths of the adjacent side and we need to find the opposite side, we will use the tan function (TOA).

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

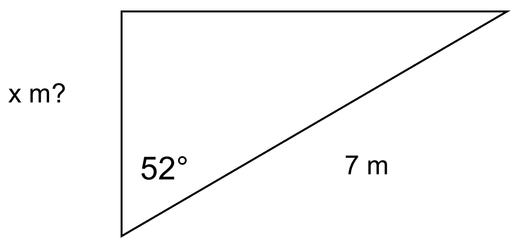
Plug in what you know:

$\tan(30°) = x/20$ (Use algebra to solve for x)

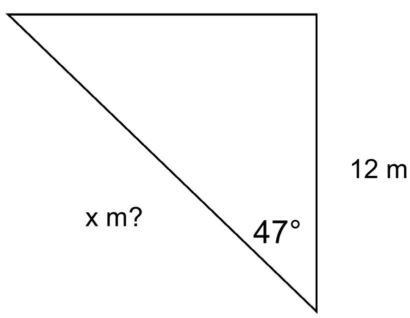
$x = 20 \tan(30°)$
$x = 11.55$ m (Solution)

Sample Problems

10.



11.



Solution

We are asked to find the length of one of the shorter sides. They give us the length of the hypotenuse and an interior angle. The side in question is adjacent to the angle. Remember SOH-CAH-TOA. Since we are given the lengths of the hypotenuse and we need to find the adjacent side, we will use the cos function (CAH).

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Plug in what you know:

$\cos(52°) = x / 7$

$x = 7 \cos(52°)$
$x = 4.31$ m (Solution)

We are asked to find the length of the hypotenuse. They give us the length of one of the shorter sides and an interior angle. The side that is marked as 12m is adjacent to the angle. Remember SOH-CAH-TOA. Since we are given the lengths of the adjacent side and we need to find the hypotenuse, we will use the cos function (CAH).

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Plug in what you know:

$\cos(47°) = 12 / x$

$x = 7 / \cos(47°)$
$x = 10.26$ m (Solution)

Scalars and Vectors

In physics, some numbers are scalars and some are vectors. Learning to work well with vectors will save you a lot of time and work. It's not too difficult really.

A **scalar** is a number that denotes a certain magnitude, or "bigness".
A **vector** is a number that denotes a certain magnitude, or "bigness" **and** gives direction.

Some common scalars are:
Mass (kg); Temperature (K); Time (s)

Some common vectors are:
Velocity (m/s); Acceleration (m/s^2);

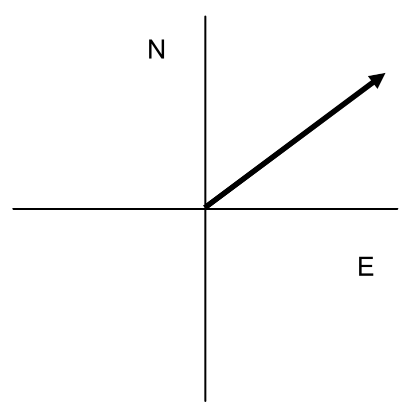
If you are ever unsure whether a number is a vector or a scalar, assume it's a vector and then draw a picture and think about it a little more.

Vector Anatomy

This is a scalar (no direction) | This is a vector (has direction)

Scalars are like cats, they lay around and don't do anything.
Vectors are like dogs, they can do all sorts of tricks.

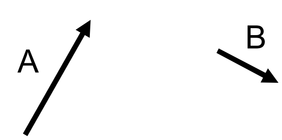
Notice, the vector tail goes to the origin of the graph, it's head pointing in its direction of motion. This vector could represent a bird flying north east.

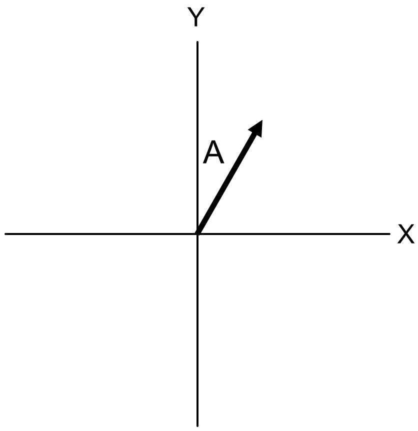


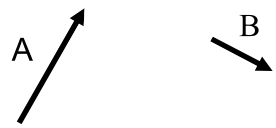
Vectors can be broken down into two parts called components. Each vector has a x and y component, which we will discuss later.

Vector Addition

Suppose you have two vectors, A and B. You can see each has a certain length (these two aren't equal) and a direction.
When two or more vectors are added together, you get another vector.

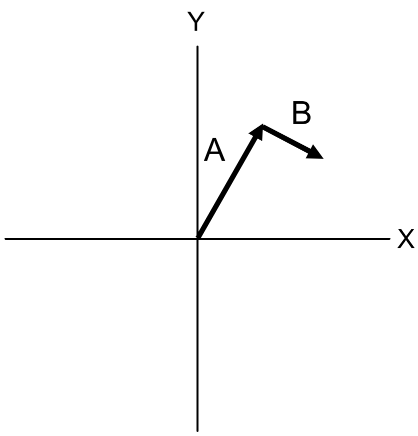




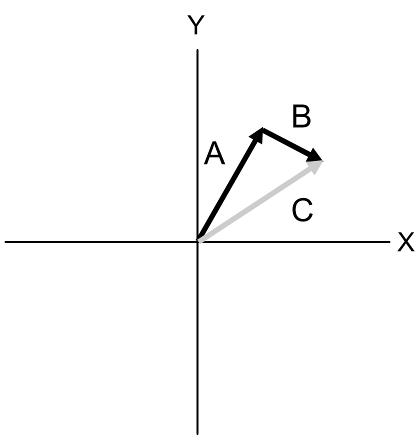


Here's our two vectors, A&B

Step 1.
Draw your first vector (pick either) on the grid with it's tail end on the origin.

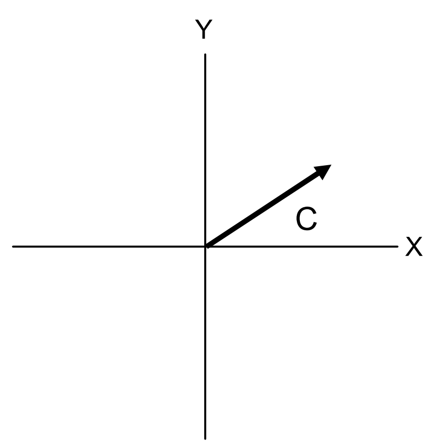


Step 2.
Put your **second vector's tail on the first vectors head.** (Do the same for third and forth vectors too if there are any.)



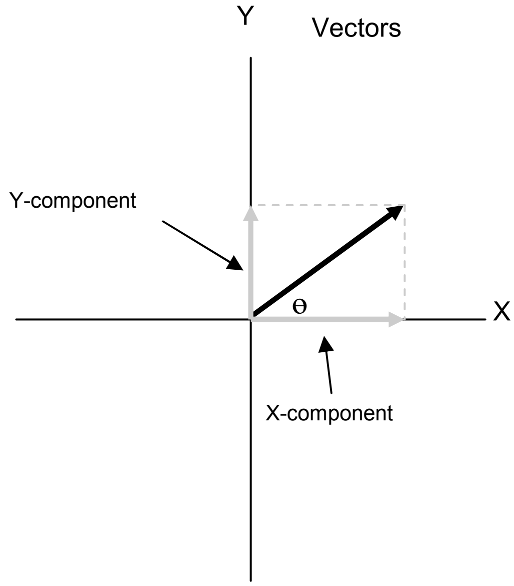
Step 3.
Now draw a new vector, "C", starting with a tail at the origin and pointing to the head of the last vector you drew. In our case, vector "B". Vectors A+B=C.

In Physics 105, you will only work with vectors in 2 dimensions, Often times your grids/graphs will have axes:
–X to X, -Y to Y
West to East, South to North
Left to Right, Down to Up
Because you only work in two dimensions, this means you can use your trigonometry skills to break each vector up into two **components.** (that's next)



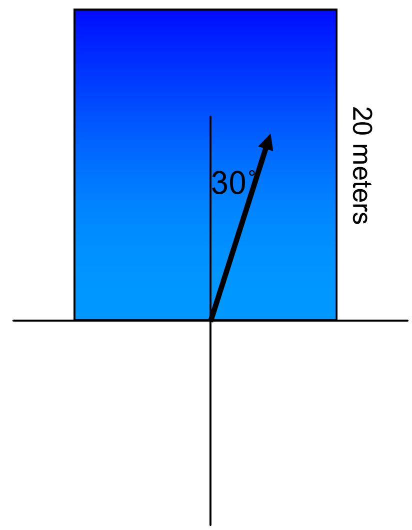
Vector Components (THIS IS VERY IMPORTANT)

This graph (at right) shows how a single vector can be broken up into it's two components, X and Y. The darker vector is our original vector. It's components tell us how much the vector changes in the X or Y direction.



Steve the swimmer

Steve likes to swim, but he always goes crooked. His friends tell him it's because he only does right-handed arm curls in the weight room and his disproportioned muscles are to blame...regardless, Steve is going for the new school record! Swimming at 5 m/s but 30 degrees off course, how long does it take Steve to swim across the 20 meter pool?



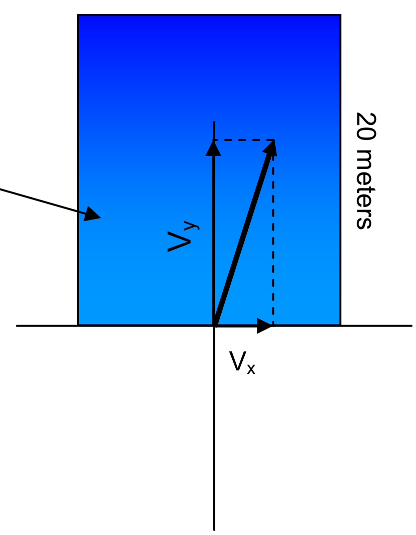
It's obvious Steve isn't taking the shortest path possible, also, all his energy isn't moving towards one goal. When Steve strokes forward, he moves forward **and** right.

The use of vector components is necessary to solve this problem.

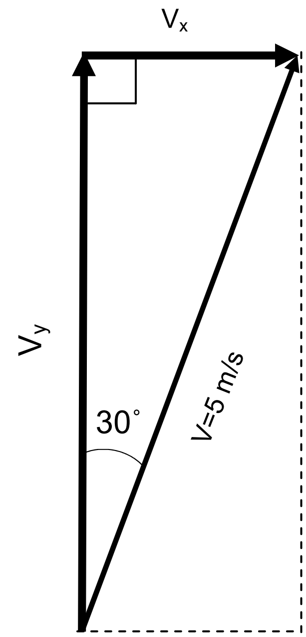
Velocity = Distance/ Time (v=d/t). To find how long it takes Steve to cross the pool, we solve for time and get t=d/v.

How long is Steve's swim? That's what we want to know
How far is Steve's cross-pool swim? 20 meters
What is his velocity <u>across</u> the pool? Hrm…

Check out the vector components of Steve's swim. Notice that Steve's 5m/s swimming velocity is the combination two smaller vectors. It is as if Steve swims in the X-direction and in the Y-direction at the same time. This is what a diagonal is, a combination of two directions. Call these two new components V_y and V_x. (velocity in the Y-direction, and velocity in the X-direction).



So what is Steve's velocity? V_y (Velocity in the Y-direction). We use V_y because we want to know how long it takes to cross the pool. The given speed of 5m/s tells us his speed moving right and forward, but we need to know how fast Steve is moving in the forward direction (Y-direction) and that direction only.



This is the Vector (and its components) representing Steve's swimming velocity (5 m/s). Notice, by moving the Vx vector from the bottom to the top, all angles and lengths are preserved but now we have a nice **right triangle**. Right triangles are your friend because they allow you to use all your trigonometry formulas.

We want to find Vy. Knowing the hypotenuse (V=5 m/s) how can we find it? Vy is adjacent to the 30degree angle given in the problem.

Using

$$\cos(\theta) = \frac{adj}{hyp} \Rightarrow \cos(\theta) = \frac{Vy}{V};$$

This becomes:

$$Vy = \cos(\theta)\ V \Rightarrow Vy = 5\cos(30)$$

Evaluating we find Vy:=4.33 m/s

Now, to solve the problem:

How long is Steve's swim? That's what we want to know
How far is Steve's cross-pool swim? 20m
What is his velocity **<u>across</u>** **the pool?** 4.33 m/s

$$v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$$

d=20m; v=4.33; solving for t, we find **it takes Steve 4.62 seconds to cross the pool.**

Now suppose we wanted to find how far off course Steve was when the reached the other side of the pool.

First, we solve for Vx:

$$\sin(\theta) = \frac{opp}{adj} \Rightarrow \sin(\theta) = \frac{Vx}{V}$$

Therefore:

$$Vx = V\cdot\sin(\theta) \Rightarrow Vx = 5\sin(30)$$

Solving, Vx=2.5m/s

From the first part, we know Steve's swim took 4.33 seconds start to finish. For 4.33 seconds, Steve was swimming 2.5m/s to the right.

$$d = v\ t$$

v=2.5m/s; t=4.33s; solving for d, we find Steve swam 10.83 meters off course

REMEMBER:

1. To find the components of a triangle, **Vx, Vy, and V must have the same units!** Always!
2. Vx and Vy are **aways** smaller than V.
3. If you have a vector at an angle you will rarely use it except to find Vx and Vy.

Dimensional Analysis

Dimensional analysis is a tool used to change units. Sometimes we are given information in one kind of units, but we are asked to give the answer in different units. For example, we might be given how many feet someone travels in a certain amount of time. We might be asked what the average speed is in miles per hour (MPH). In this case, you would need to know the conversion between feet and mile. You would have to know that there are 5280 feet in one mile. The necessary conversion factors can be found in your textbook. The following example illustrates converting given units into the units that we are asked for:

Example:

Steve bought a car without a speedometer (Steve's going through a rough time). Oddly enough, it has an instrument that measures how many feet the car travels. Steve also has a nice stop watch. He wants to find his average speed in MPH on the way to his friend's house. He finds that it took him 480 seconds to go 21,120 feet. What is his average MPH?

Solution

We are given the information in feet and seconds. We want to get the answer in miles per hours. Our task will be to convert feet into miles and seconds into hours and divide.

$$\text{Average speed} = \frac{21{,}120 \text{ feet}}{480 \text{ seconds}}$$

Questions to ask

How many feet are in one mile?
5,280 feet
How many seconds are in one minute?
60 seconds
How many minutes are in one hour?
60 minutes

We can't change the original information, but we can modify it by multiplying by one. If one mile = 5,280 feet, then $\frac{\text{1 mile}}{\text{5,280 feet}} = 1$. If we multiply anything by this fraction then we won't change anything. Our objective is to change units. So if we multiply our first equation that we have from the given information by $\frac{\text{1 mile}}{\text{5,280 feet}}$ then we will get miles divided by seconds or miles per second.

$$\text{Average speed} = \frac{\text{21,120 feet}}{\text{480 seconds}} \times \frac{\text{1 mile}}{\text{5,280 feet}}$$ (Since feet are on the top and bottom they cancel out)

$$\text{Average speed} = \frac{21{,}120 \times \text{1 mile}}{\text{480 seconds} \times 5{,}280} = \frac{\text{4 miles}}{\text{480 seconds}}$$

Now we have our answer in miles/seconds or miles per seconds. We need to convert the seconds into hours. We know that there are 60 seconds in one minute and 60 minutes in one hour. We will use those relationships to convert seconds into hours.

$$\text{Average speed} = \frac{\text{4 miles}}{\text{480 seconds}} \times \frac{\text{60 seconds}}{\text{1 minute}}$$

$$\text{Average speed} = \frac{\text{4 miles} \times 60}{480 \times \text{1 minute}} = \frac{\text{240 miles}}{\text{480 minutes}} \overset{\text{simplifying}}{=} \frac{\text{1 mile}}{\text{2 minutes}}$$

$$\text{Average speed} = \frac{\text{1 mile}}{\text{2 minutes}} \times \frac{\text{60 minutes}}{\text{1 hour}} = \frac{\text{1 mile} \times 60}{2 \times \text{1 hour}} = \frac{\text{60 miles}}{\text{2 hours}}$$

$$\text{Average speed} = \frac{\text{30 miles}}{\text{1 hour}} = \text{30 MPH}$$ There's your answer!

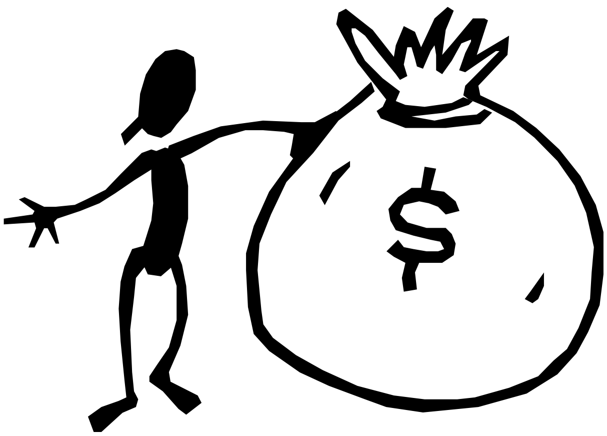
You can do the whole preceding process in one step. If you know how to get from the units that you have been given to the ones that were asked for you can include them all in one giant equation like so:

$$\text{Average speed} = \frac{\text{21,120 feet}}{\text{480 seconds}} \times \frac{\text{1 mile}}{\text{5,280 feet}} \times \frac{\text{60 seconds}}{\text{1 minute}} \times \frac{\text{60 minutes}}{\text{1 hour}} = \frac{\text{30 miles}}{\text{1 hour}}$$
$$= \text{30 MPH}$$

To get the answer you simply multiply across the top and divide by the number you get by multiplying across the bottom.

Sample Problems

1. Steve travels over to England with a big bag of money. Once there he realizes that they only take British Pounds. He decides to go to the exchange house to change his money into Pounds. He counts the money in his bag and finds that he has $526 (all in dollar bills). The current exchange rate is $1.99 American Dollars to 1 British Pound. How many British Pounds should he get?



Solution

We are given information in US dollars and we are asked to convert it into British Pounds. We are given the conversion rate.

$1.99 = £1.00(Pound)

Therefore $\frac{£1.00}{\$1.99} = 1$

That means that we can multiply anything by the conversion factor and it wouldn't change anything. We want to multiply so we cancel out $'s and we are left with £'s. Let's multiply $526 by the conversion factor.

$$\$526 \text{ x } \frac{£1.00}{\$1.99} = \frac{\$526 \text{ x } £1.00}{\$1.99}$$

(Dollars cancel out because they are on the top and bottom)

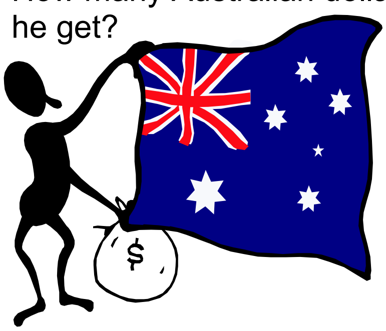
$$= \frac{£526}{1.99}$$

$$= £264.32$$

You can do it all in one step

$$\$526 \text{ x } \frac{£1.00}{\$1.99} = £264.32$$

2. Steve heads down under to Australia after England. He had decided not to convert all of his US dollars into pounds, so he's left with a small bag of US dollars. He decides to exchange the rest of his money to Australian dollars. He has 67 US dollars left. The current exchange rate is 1 Australian dollar = .83 US dollars. How many Australian dollars should he get?



We are given information in US dollars and we are asked to convert it into Australian dollars. We are given the conversion rate.

0.83 US dollars = 1.00 Australian Dollars

Therefore

$$\frac{1.00 \text{ AUD (Australian dollar)}}{1.99 \text{ USD (US dollar)}} = 1$$

(conversion factor)

Again that means that we can multiply anything by the conversion factor and it wouldn't change anything. We want to multiply so we cancel out USD's and we are left with AUD's. Let's multiply $526 by the conversion factor.

Sample Problems

2. Continued

Solution

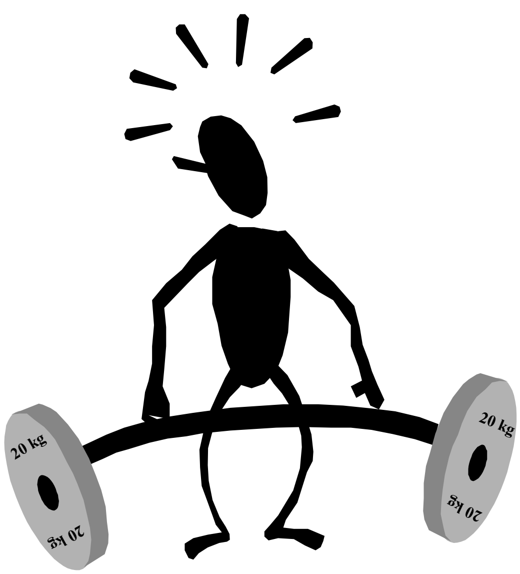
$= \frac{67\ \text{AUD}}{0.83}$

$= 80.72\ \text{AUD}$

Again we can do it all in one step.

$67\ \text{USD} \times \frac{1.00\ \text{AUD}}{0.83\ \text{USD}} = 80.72\ \text{AUD}$

3. While in Australia, Steve works out at the gym. All the weights are marked in kilograms instead of pounds, like he's used to. Steve can only curl 70 pounds. He finds a bar with weights that are marked 20 kg on each side. Can he curl these weights? (1 kg = 2.2 lbs).



We are given information in kilograms and we are asked to convert it into pounds. We are given the conversion rate.

2.2 lbs. = 1.00 kg.

Therefore

$\frac{1.00\ \text{kg}}{2.2\ \text{lbs.}} = 1$ (conversion factor)

Again that means that we can multiply anything by the conversion factor and it wouldn't change anything. We want to multiply so we cancel out lbs. and we are left with kg. Let's multiply 40 kg. (20 kg. x 2) by the conversion factor.

$40\ \text{kg} \times \frac{2.2\ \text{lbs.}}{1.00\ \text{kg.}} = \frac{40\ \text{kg} \times 2.2\ \text{lbs.}}{0.83\ \text{kg}}$

$= \frac{88\ \text{lbs.}}{1.00}$

$= 88\ \text{lbs.}$

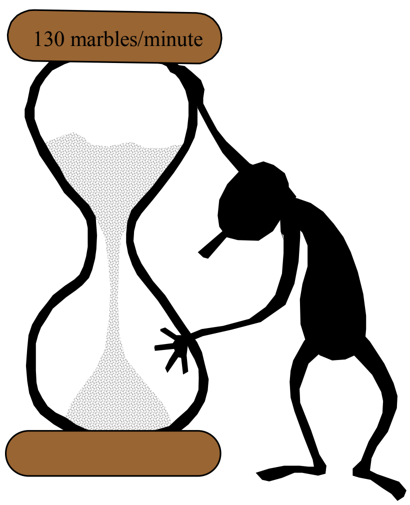
One step process

$40\ \text{kg} \times \frac{2.2\ \text{lbs.}}{1.00\ \text{kg.}} = 88\ \text{lbs.}$

He can't lift it.

Sample Problems

4. Steve lost his stopwatch. He wants to time how fast his friend can run around the block. He has a giant hour glass in his house. Instead of sand, it's full of marbles. The hour glass is marked with 130 marbles/minute. He starts the timer when his friend leaves and then puts it on its side when he finishes and then counts the marbles in the lower compartment. If he counts 325 marbles, how long did it take his friend to run around the block?



Solution

We are given information in a quantity of marbles and we are asked to convert it into time or minutes. We are given the conversion rate.

130 marbles = 1 minute

Therefore

$$\frac{130 \text{ marbles}}{1 \text{ minute}} = 1$$

That means that we can multiply anything by the conversion factor and it wouldn't change anything. We want to multiply so we cancel out marbles and we are left with minutes. Let's multiply 325 marbles by the conversion factor.

$$325 \text{ marbles} \times \frac{1 \text{ minute}}{130 \text{ marbles}}$$

$$= \frac{325 \text{ marbles} \times 1 \text{ minute}}{130 \text{ marbles}}$$

$$= \frac{325 \text{ minutes}}{130}$$

$$= 2.5 \text{ minutes}$$