

**H17.** Ch10: 2,4,26,28,33,34,41 (This looks like a lot, but some are simple, some with help in back)

**PB.** Array of circular holes.

1. For four holes of diameter  $l$  on the edges of a square of length  $d$ , write the Fraunhofer diffraction

pattern  $I(\theta_y, \theta_z)$  using the array theorem and  $I(\theta) = I_o \left[ \frac{J_1(kl\theta/2)}{(kl\theta/2)} \right]^2$  for a single hole. You will

have to replace the cylindrical angle  $\theta$  by  $\theta = \sqrt{\theta_y^2 + \theta_z^2}$  to use Cartesian variables  $\theta_y, \theta_z$ .

2. Plot  $I(\theta_y, \theta_z)$  for a single hole for the case  $kl/2 = 1, I_o = 1$ . You could use

>> `with(plots); plot3d(I_aper(tx,ty)*I_array(tx,ty),tx=-5..5,ty=-5..5,grid=[50,50]);`  
 tx, ty stand for theta-x, etc. Change the ranges to fit the plots. Use the first order Bessel function `BesselJ(1, x)` for the aperture.

a. Plot  $I(\theta_y, \theta_z)$  for two holes on the bottom edge of the square for  $d = 3l$

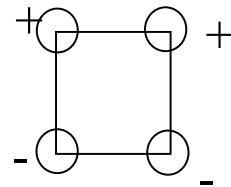
b. Plot  $I(\theta_y, \theta_z)$  for two holes across a diagonal of the square

c. Plot  $I(\theta_y, \theta_z)$  for three holes on corners of your choice

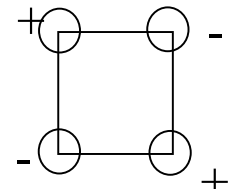
d. Plot  $I(\theta_y, \theta_z)$  for all four holes

e. Summarize what you've seen in terms of the ideas of interference and the array theorem

f. Plot  $I(\theta_y, \theta_z)$  for all four holes, but with E in each hole having the phases given at right: + means no phase change at aperture, - means  $\pi$  (180 degrees) phase shift, so multiply that contribution by -1.



g. Plot  $I(\theta_y, \theta_z)$  for all four holes, but with E in each hole having the phases given at right.



h. If instead of four holes, we had 300 holes spaced at random positions all illuminated evenly, what would the approximate  $I(\theta_y, \theta_z)$  be?

**H18.** None. Continue with H19

**H19. Ch 12.9**

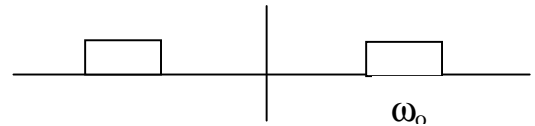
**P19A.** The nearest star (other than our sun) to us is Proxima Centauri at a distance of 30 trillion kilometers, and it has an angular diameter of 2 millionth of a degree or 7 milliarcseconds (1 milliarcsecond is 1 thousandth of an arcsecond which is one sixtieth of an arcminute which is one sixtieth of a degree...thanks to the Babylonians, who loved multiples of 60).

1. If an astronomer filters the white starlight through a green filter,  $\lambda=500$  nm,  $\Delta\lambda = 50$  nm, what are the approximate temporal coherence length and time of the light after passing through the filter?
2. Using concepts from class, sketch the power spectrum  $I(\omega)$  for this light, and label the approximate width in rad/sec, as well as the position of the peak in rad/sec.
3. Using concepts from class, sketch the interferogram  $I(\tau)$  for delay  $\tau$  for this single star. Show the approximate width of the wiggles in femtoseconds (fs), as well as the approximate coherence time.
4. What is the approximate spatial coherence length of this light due to the angular width of the star? Use a single  $\lambda = 500$  nm for this analysis, and the simple estimates we did in class. This length is approximately the baseline of a stellar interferometer needed to resolve the star's diameter.

**P19B.** With some filters, we arrange a square (“shutter-like”) spectrum  $I(\omega)$  centered around  $\pm\omega_o$ :

$$I(\omega) = 1 \quad -\left(\omega_o + \frac{\Delta\omega}{2}\right) < \omega < -\left(\omega_o - \frac{\Delta\omega}{2}\right) \quad \text{and} \quad \omega_o - \frac{\Delta\omega}{2} < \omega < \omega_o + \frac{\Delta\omega}{2}$$

$$I(\omega) = 0 \quad \text{elsewhere}$$



1. Find  $\gamma(\tau)$  for this light using 
$$\gamma(\tau) \equiv \frac{\int_{-\infty}^{\infty} d\omega I(\omega) e^{i\omega\tau}}{\int_{-\infty}^{\infty} I(\omega) d\omega}$$

2. Plot the Michelson interferogram  $I(\tau)$  for this light, for  $\omega_o = 1$  and  $\Delta\omega = \frac{\omega_o}{50}$ . Plot it long enough to see the visibility go to zero and a bit beyond.

3. Find the coherence time for the same conditions ( $\omega_o = 1$  and  $\Delta\omega = \frac{\omega_o}{50}$ ) from a numerical integration:

$$\tau_c \equiv \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau.$$

4. How should the coherence time relate in an approximate way to  $\Delta\omega$ ? Check this for your results in 3). How many periods of oscillation is one coherence time in this case?

5. Looking at your plotted  $I(\tau)$ , calculate the visibility at  $\frac{\tau_c}{2}$  and  $\tau_c$ . Calculate  $|\gamma(\tau)|$  at those delays as well. How do they compare?

**P19C.** Suppose we have a star of angular radius  $\theta_{star}$ , whose intensity is maximum in the center, and goes linearly to zero at  $\theta_{star}$ . This will be a “triangular intensity” profile, as opposed to a square one.

1. Find visibility  $V(a) = \left| \gamma(a) \right| = \left| \frac{\int_{-\infty}^{\infty} I(\theta') e^{-ika\theta'} d\theta'}{\int_{-\infty}^{\infty} I(\theta') d\theta'} \right|$  in terms of  $a$ ,  $\theta_{star}$  and  $\lambda$ . Note:  $\int_{-\infty}^{\infty} x e^{bx} dx = \frac{e^{bx}}{b^2} (bx - 1)$

2. Plot the visibility  $V(a)$  for  $\lambda = 500$  nm, and the same angular radius of problem 1. You will want to plot it over distances near to the baseline you estimated in problem A. Define the spatial coherence length as the smallest value of  $a$  that gives a zero visibility, and compare to your estimate in problem A.

**H20** 13.21 (take 3.0mm as the beamwaist), 29, 30, 31, 35

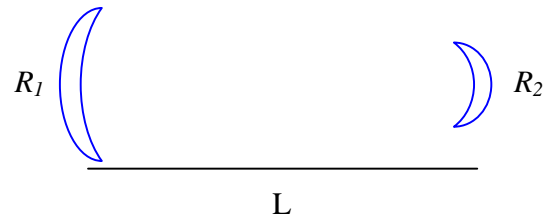
**P20A.** Longitudinal laser modes.

1. Derive the simple relation between longitudinal mode number and allowed frequency  $\nu_m$  given integer half-wavelengths fit between the two mirrors. For a HeNe laser,  $\lambda=632$  nm, find the approximate  $m$ -value when  $L=0.5$ m.
2. If the gain bandwidth is  $\Delta\nu=1.5$  GHz FWHM (from Doppler broadening) find how many longitudinal modes are lasing. Find the coherence length of the light that is emitted.
3. When the HeNe is stabilized with a grating or other optics so that only one longitudinal mode is allowed to lase, the width of this single- $m$  line is determined by the leakage time of light from the cavity:

$\Delta\nu_{\text{line}} = \frac{c}{2L}(1-R)$ , where  $R$  is the reflectance of the output coupler. Use  $R = 99.0\%$ . Calculate  $\Delta\nu_{\text{line}}$  and estimate the coherence length of the light that is emitted when only one line lases.

**P20B.**

1. For a laser cavity with  $R_1 = 2R$ ,  $R_2 = R$ , and  $L = 5/2 R$ , find the location of the beamwaist measured from  $R_2$ , and the beamwaist in terms of  $R$  and  $\lambda$ . Note that you have three unknowns: distances of the beamwaist from each mirror, and



2. Evaluate  $z_o$ ,  $w_o$ , the location of the beamwaist, and the divergence angle of this laser for  $R = 20$  cm and  $\lambda = 600$  nm.

**P20C.**

1. For a 5mW HeNe laser ( $\lambda=632$  nm), find the energy density  $u_{\text{beam}}$ , if it has a beam radius of 0.5mm. (Remember  $I = uc$ ).
2. Inside the cavity,  $I$  is greater because only a fraction is transmitted by the output coupler to form the output beam:  $\frac{I_{\text{cavity}}}{2}(1-R_{\text{coupler}}) = I_{\text{beam}}$  (the factor of 2 relates to the fact that we have beams in both direction in the cavity). Find  $u_{\text{cavity}}$  from this relation if  $R_{\text{coupler}} = 0.99$ .
3. Using this  $u_{\text{cavity}}$ , find the ratio of the stimulated emission rate compared to the spontaneous emission rate from Einstein's relations. (Note this assumes all the light is in one mode, not generally true, but we we'll go with it.)