

Homework Assignments

HW1.

Ch2: 32, 37, 40, 42 (note problems without * have solutions in back...but don't look until you've tried it, or your learning will go way down)

and:

Problem 1A:

$$\psi_1 = (3 + 2i)e^{i(2x-4t)}$$

$$\psi_2 = (-1 + 4i)e^{i(2x-4t)}$$

1. Find ω and λ
2.
 - a. Represent each wave and their sum on a phasor diagram for $x, t = (0, 0)$
 - b. Find analytically the magnitude and phase of $\psi_1 + \psi_2$
3. Repeat 2) with $x, t = (1, 2)$

Problem 1B: Using Maxwell's equations (derivative form), and the forms

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = (E_{ox} \hat{x} + E_{oy} \hat{y} + E_{oz} \hat{z}) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = (B_{ox} \hat{x} + B_{oy} \hat{y} + B_{oz} \hat{z}) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

show that $\vec{\nabla} \times \vec{E} = \vec{k} \times \vec{E}$. Then show it follows that $\vec{k} \times \vec{E} = \omega \vec{B}$ and $\vec{k} \times \vec{B} = -\mu_0 \epsilon_0 \omega \vec{E}$.

From this, argue that: E, B must be perpendicular to k vector and to each other, and $B_o = \frac{E_o}{c}$

HW2 on next page

HW2

Ch3: 5,18,29,36,40

and

Problem 2A:

Write an E-field in the form $\vec{E} = \vec{E}_o e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ polarized in the y-direction, moving in the x direction, and delayed in time by one quarter cycle relative to the oscillation $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

Problem 2B:

Suppose you claim $\vec{E} = \hat{k}(x^2 + y^2 + z^2) \cos \omega t$. From Maxwell's equations, find the charge density $\rho(x, y, z, t)$ that would have to accompany it. Find $\vec{\nabla} \times \vec{E}$ and from it $\vec{B}(x, y, z, t)$. Show that this B violates a third Maxwell equation, so the $\vec{E}(x, y, z, t)$ you proposed is not physical.

Problem 2C:

In eq 3.66, $x(t) = x_o \cos \omega t$ inserted into the differential equation 3.64 yields $x_o = \frac{qE_o}{m(\omega_o^2 - \omega^2)}$ This is very

simple. But damping is missing, so it's not physical because the amplitude goes to infinity at resonance. If

we add damping γ , the differential equation is $\frac{\partial^2 x}{\partial t^2} + \gamma \frac{\partial x}{\partial t} + \omega_o^2 x = \frac{q}{m} E_o \cos \omega t$. This is a pain to solve with

cos and sin. Instead, substitute $x(t) = x_o e^{i\omega t}$ into $\frac{\partial^2 x}{\partial t^2} + \gamma \frac{\partial x}{\partial t} + \omega_o^2 x = \frac{q}{m} E_o e^{i\omega t}$. Show that x_o is proportional

to $\frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega}$. Break this factor into its real and imaginary parts and plot them: set $\omega_o = 1$, and plot (for

$\omega = 0$ to 3) the curves for three different dampings that you choose to illustrate what happens as γ changes.

Notice which part (real or imaginary) has the spectral shape expected for of index $n(\omega)$, and which has the shape of an absorption curve. Note that the real and imaginary parts correspond to the cos and sin parts of the (harder) trigonometric solution.

HW3

Ch. 4: 2,7,8,26, 27

Addenda: On 8, show that regardless of the incident angle with respect to the first normal line drawn, the incoming and outgoing lines are parallel, which is why surveyors use corner mirrors for their laser beam reflectors. On 27, relate the proof here to the photon momentum conservation discussion in 4.11.2

Problem 3A

In the Lorentz model, take $N = 10^{28}/\text{m}^3$ for the density of bound electrons in an insulator, and a single transition at $\omega_0 = 6 \times 10^{15}$ rad/sec (in the UV), and damping $\gamma = \omega_0/5$ (quite broad). Note this is the example of the graphs I presented in class, so you can check your value of n .

For the frequency $\omega = \omega_0 - 2\gamma$:

- i. Find the relative dielectric constant $K(\omega)$: its magnitude and phase. Does this lead or lag $E(t)$?
 - ii. find $\tilde{n} = n + in_i$. Please do this step by the method $\sqrt{z} = \sqrt{|z|} (e^{i\theta})^{1/2}$ so you are familiar with taking square roots of complex numbers.
 - iii. Find the speed of light in terms of c , and λ in the vacuum and in the medium
 - iv. Find the absorption coefficient α in units of 1/cm. Find how far (in cm) the light penetrates into the material before only 1/10 of the original intensity I remains.
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HW4

Ch. 4: 19, 21, 23, 24, 32, 35

Addenda: On 21, prove the general case for apparent depth given any n . Use $n_w = 1.33 = 4/3$

HW5

Problem 5A Show analytically (using trig relations and the Fresnel equations) for s-polarized light that $R_s + T_s = 1$.

Problem 5B Microwaves ($\lambda_{\text{vac}} = 3\text{cm}$) in wax ($n = 1.5$) incident at 45° on a wax-air interface are totally internally reflected. Find the decay constant α (in cm^{-1}) of the evanescent waves for both s- and p-polarization. You will measure this in the lab in HW6, so save a copy of your work so you can compare with experiment.

Problem 5C

Metal reflection, plasma. In the Lorentz model, take $N = 10^{28}/\text{m}^3$ for the density of free electrons in a metal. Assume no bound electrons, and set $\omega_o = 0$. Note this is my example in the graphs in lecture, so you can check your work.

1. Find the plasma frequency. (note: Remember this is not a bound electron resonance.)

For $\omega = \omega_p - 1 \times 10^{15} \text{ rad/sec}$:

- Find $\tilde{n} = n + in_i$ for $\gamma = 0$, and in Fresnel's equations, for normal incidence, show that with this \tilde{n} , $r^*r = R = 1$. Find the phase shift of the reflected light vs the incident light, which is the phase of r .
- Find the decay length $1/\alpha$ (for light energy to penetrate a typical distance of $1/e$) before it is reflected perfectly.
- With damping $\gamma = 1 \times 10^{15} \text{ rad/sec}$, find \tilde{n} , and for normal incidence show that $r^*r = R < 1$. This represents the loss of energy due to damping. Find the phase shift of the reflected light vs the incident light, which is the phase of r .
- Find the decay length $1/\alpha$ (for light energy to penetrate a typical distance of $1/e$). This decay represents a combination of strong reflection and some absorption. The light should penetrate less far.

For $\omega = \omega_p + 1 \times 10^{15} \text{ rad/sec}$:

- Find $\tilde{n} = n + in_i$ for $\gamma = 0$, and in Fresnel's equations, for normal incidence, and show $R + T = 1$.

The metal is a transparent material at this ω .

- With damping $\gamma = 1 \times 10^{15} \text{ rad/sec}$, find the decay length $1/\alpha$ (for light energy to penetrate a typical distance of $1/e$) due to absorption. The metal above the plasma frequency is now acting like a typical dielectric with some absorption.
- For the case in f), find the phase shift of the transmitted light vs the incident light, which is the phase of t .