PROBLEMS

Complete solutions to all problems—except those with an asterisk—can be found in the back of the book.

2.1 How many "yellow" lightwaves ($\lambda = 580$ nm) will fit into a distance in space equal to the thickness of a piece of paper (0.003 in.)? How far will the same number of microwaves ($\nu = 10^{10}$ Hz, i.e., 10 GHz, and $v = 3 \times 10^8$ m/s) extend?

2.2* The speed of light in vacuum is approximately 3×10^8 m/s. Find the wavelength of red light having a frequency of 5×10^{14} Hz. Compare this with the wavelength of a 60-Hz electromagnetic wave.

2.3* It is possible to generate ultrasonic waves in crystals with wavelengths similar to light (5×10^{-5} cm) but with lower frequencies (6×10^8 Hz). Compute the corresponding speed of such a wave.

2.4* A youngster in a boat on a lake watches waves that seem to be an endless succession of identical crests passing with a half-second interval between each. If every disturbance takes 1.5 s to sweep straight along the length of her 4.5-m-long boat, what are the frequency, period, and wavelength of the waves?

2.5* A vibrating hammer strikes the end of a long metal rod in such a way that a periodic compression wave with a wavelength of 4.3 m travels down the rod's length at a speed of 3.5 km/s. What was the frequency of the vibration?

2.6 A violin is submerged in a swimming pool at the wedding of two scuba divers. Given that the speed of compression waves in pure water is 1498 m/s, what is the wavelength of an A-note of 440 Hz played on the instrument?

2.7* A wavepulse travels 10 m along the length of a string in 2.0 s. A harmonic disturbance of wavelength 0.50 m is then generated on the string. What is its frequency?

2.8* Show that for a periodic wave $\omega = (2\pi/\lambda)v$.

2.9* Make up a table with columns headed by values of θ running from $-\pi/2$ to 2π in intervals of $\pi/4$. In each column place the corresponding value of $\sin\theta$, beneath those the values of $\cos\theta$, beneath those the values of $\sin(\theta - \pi/4)$, and similarly with the functions $\sin(\theta - \pi/2)$, $\sin(\theta - 3\pi/4)$, and $\sin(\theta + \pi/2)$. Plot each of these functions, noting the effect of the phase shift. Does $\sin\theta$ lead or lag $\sin(\theta - \pi/2)$. In other words, does one of the functions reach a particular magnitude at a smaller value of θ than the other and therefore lead the other (as $\cos\theta$ leads $\sin\theta$)?

2.10* Make up a table with columns headed by values of kx running from $x = -\lambda/2$ to $x = +\lambda$ in intervals of x of $\lambda/4$—of course, $k = 2\pi/\lambda$. In each column place the corresponding values of $\cos(kx - \pi/4)$ and beneath that the values of $\cos(kx + 3\pi/4)$. Next plot the functions $15\cos(kx - \pi/4)$ and $25\cos(kx + 3\pi/4)$.

2.11* Make up a table with columns headed by values of ωt running from $t = -\tau/2$ to $t = +\tau$ in intervals of t of $\tau/4$—of course, $\omega = 2\pi/\tau$. In each column place the corresponding values of $\sin(\omega t + \pi/4)$ and $\sin(\pi/4 - \omega t)$ and then plot these two functions.

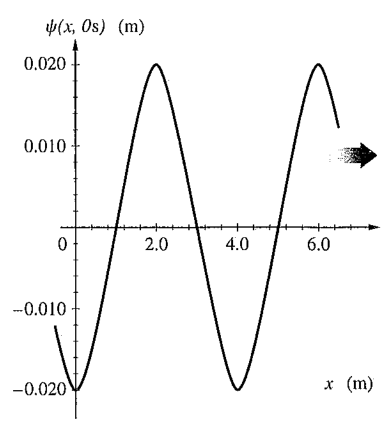
2.12* The profile of a transverse harmonic wave, traveling at 1.2 m/s on a string, is given by

$$y = (0.02\text{ m})\sin(157\text{ m}^{-1})\,x$$

Determine its amplitude, wavelength, frequency, and period.

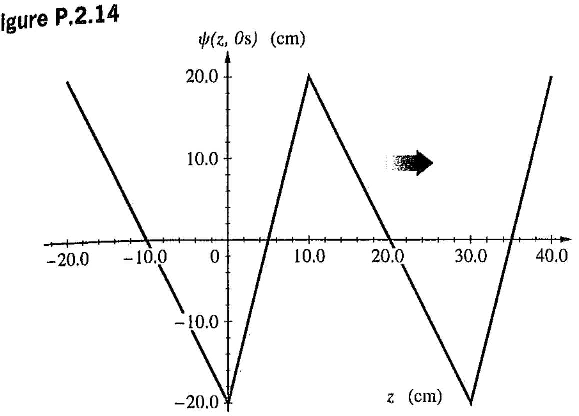
2.13* Figure P.2.13 represents the profile ($t = 0$) of a transverse wave on a string traveling in the positive x-direction at a speed of 20.0 m/s. (a) Determine its wavelength. (b) What is the frequency of the wave? (c) Write down the wavefunction for the disturbance. (d) Notice that as the wave passes any fixed point on the x-axis the string at that location oscillates in time. Draw a graph of the ψ versus t showing how a point on the rope at $x = 0$ oscillates.

Figure P.2.13



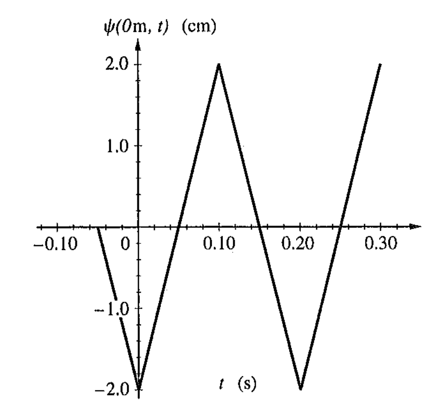
2.14* Figure P.2.14 represents the profile ($t = 0$) of a transverse wave on a string traveling in the positive z-direction at a speed of 100 cm/s. (a) Determine its wavelength. (b) Notice that as the wave passes any fixed point on the z-axis the string at that location oscillates in time. Draw a graph of the ψ versus t showing how a point on the rope at $x = 0$ oscillates. (c) What is the frequency of the wave?

Figure P.2.14



2.15* A transverse wave on a string travels in the negative y-direction at a speed of 40.0 cm/s. Figure P.2.15 is a graph of ψ versus t showing how a point on the rope at $y = 0$ oscillates. (a) Determine the wave's period. (b) What is the frequency of the wave? (b) What is the wavelength of the wave? (d) Sketch the profile of the wave (ψ versus y).

Figure P.2.15



2.16 Given the wavefunctions

$$\psi_1 = 4\sin 2\pi(0.2x - 3t)$$

and

$$\psi_2 = \frac{\sin(7x + 3.5t)}{2.5}$$

determine in each case the values of (a) frequency, (b) wavelength, (c) period, (d) amplitude, (e) phase velocity, and (f) direction of motion. Time is in seconds and x is in meters.

2.17* The wavefunction of a transverse wave on a string is

$$\psi(x, t) = (30.0\text{ cm})\cos[(6.28\text{ rad/m})x - (20.0\text{ rad/s})t]$$

Compute the (a) frequency, (b) wavelength, (c) period, (d) amplitude, (e) phase velocity, and (f) direction of motion.

2.18* Show that

$$\psi(x, t) = A\sin k(x - vt) \qquad [2.13]$$

is a solution of the differential wave equation.

2.19* Show that

$$\psi(x, t) = A\cos(kx - \omega t)$$

is a solution of the differential wave equation.

2.20* Prove that

$$\psi(x, t) = A\cos(kx - \omega t - \pi/2)$$

is equivalent to

$$\psi(x, t) = A\sin(kx - \omega t)$$

2.21 Show that if the displacement of the string in Fig. 2.7 is given by

$$y(x, t) = A\sin[kx - \omega t + \varepsilon]$$

then the hand generating the wave must be moving vertically in simple harmonic motion.

2.22 Write the expression for the wavefunction of a harmonic wave of amplitude 10^3 V/m, period 2.2×10^{-15} s, and speed 3×10^8 m/s. The wave is propagating in the negative x-direction and has a value of 10^3 V/m at $t = 0$ and $x = 0$.

2.23 Consider the pulse described in terms of its displacement at $t = 0$ by

$$y(x, t)|_{t=0} = \frac{C}{2 + x^2}$$

where C is a constant. Draw the wave profile. Write an expression for the wave, having a speed v in the negative x-direction, as a function of time t. If $v = 1$ m/s, sketch the profile at $t = 2$ s.

2.24* Please determine the magnitude of the wavefunction $\psi(z, t) = A\cos[k(z + vt) + \pi]$ at the point $z = 0$, when $t = \tau/2$ and when $t = 3\tau/4$.

2.25 Does the following function, in which A is a constant,

$$\psi(y, t) = (y - vt)A$$

represent a wave? Explain your reasoning.

2.26* Use Eq. (2.33) to calculate the speed of the wave whose representation in SI units is

$$\psi(y, t) = A \cos \pi(3 \times 10^6 y + 9 \times 10^{14} t)$$

2.27 Beginning with the following theorem: If $z = f(x, y)$ and $x = g(t)$, $y = h(t)$, then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Derive Eq. (2.34).

2.28 Using the results of the previous problem, show that for a harmonic wave with a phase $\varphi(x, t) = k(x - vt)$ we can determine the speed by setting $d\varphi/dt = 0$. Apply the technique to Problem 2.26 to find the speed of that wave.

2.29* A Gaussian wave has the form $\psi(x, t) = Ae^{-a(bx+ct)^2}$. Use the fact that $\psi(x, t) = f(x \mp vt)$ to determine its speed and then verify your answer using Eq. (2.34).

2.30 Create an expression for the *profile* of a harmonic wave traveling in the z-direction whose magnitude at $z = -\lambda/12$ is 0.866, at $z = +\lambda/6$ is $1/2$, and at $z = \lambda/4$ is 0.

2.31 Which of the following expressions correspond to traveling waves? For each of those, what is the speed of the wave? The quantities a, b, and c are positive constants.

(a) $\psi(z, t) = (az - bt)^2$

(b) $\psi(x, t) = (ax + bt + c)^2$

(c) $\psi(x, t) = 1/(ax^2 + b)$

2.32* Determine which of the following describe traveling waves:

(a) $\psi(y, t) = e^{-(a^2y^2+b^2t^2-2abty)}$

(b) $\psi(z, t) = A \sin (az^2 - bt^2)$

(c) $\psi(x, t) = A \sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2$

(d) $\psi(x, t) = A \cos^2 2\pi(t - x)$

Where appropriate, draw the profile and find the speed and direction of motion.

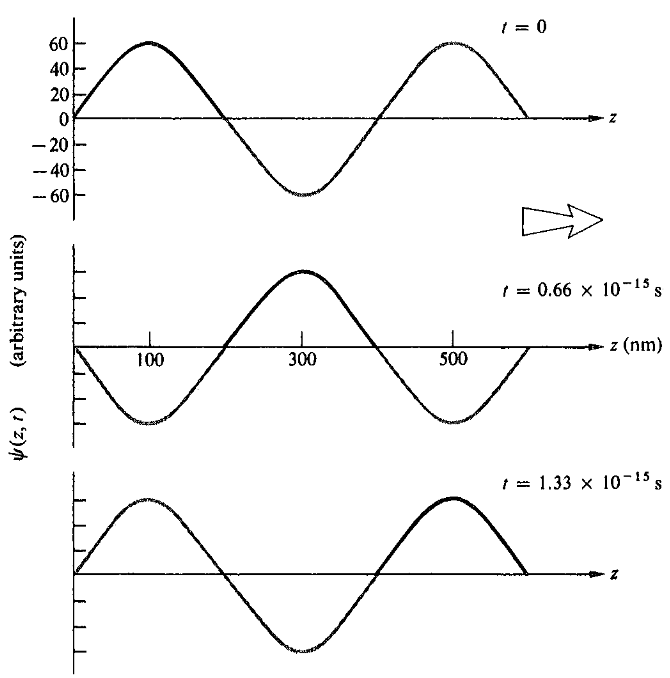
2.33 Given the traveling wave $\psi(x, t) = 5.0 \exp(-ax^2 - bt^2 - 2\sqrt{ab}\,xt)$, determine its direction of propagation. Calculate a few values of ψ and make a sketch of the wave at $t = 0$, taking $a = 25\ \text{m}^{-2}$ and $b = 9.0\ \text{s}^{-2}$. What is the speed of the wave?

2.34* Imagine a sound wave with a frequency of 1.10 kHz propagating with a speed of 330 m/s. Determine the phase difference in radians between any two points on the wave separated by 10.0 cm.

2.35 Consider a lightwave having a phase velocity of 3×10^8 m/s and a frequency of 6×10^{14} Hz. What is the shortest distance along the wave between any two points that have a phase difference of 30°? What phase shift occurs at a given point in 10^{-6} s, and how many waves have passed by in that time?

2.36 Write an expression for the wave shown in Fig. P.2.36. Find its wavelength, velocity, frequency, and period.

Figure P.2.36 A harmonic wave.



2.37* Working with exponentials directly, show that the magnitude of $\psi = Ae^{i\omega t}$ is A. Then rederive the same result using Euler's formula. Prove that $e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)}$.

2.38* Show that the imaginary part of a complex number $\tilde{z}$ is given by $(\tilde{z} - \tilde{z}^*)/2i$.

2.39 Beginning with Eq. (2.51), verify that

$$\psi(x, y, z, t) = Ae^{i[k(\alpha x+\beta y+\gamma z)\mp\omega t]}$$

and that

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Draw a sketch showing all the pertinent quantities.

2.40* Show that Eqs. (2.64) and (2.65), which are plane waves of arbitrary form, satisfy the three-dimensional differential wave equation.

2.41 De Broglie's hypothesis states that every particle has associated with it a wavelength given by Planck's constant ($h = 6.6 \times 10^{-34}$ J·s) divided by the particle's momentum. Compare the wavelength of a 6.0-kg stone moving at a speed of 1.0 m/s with that of light.

2.42 Write an expression in Cartesian coordinates for a harmonic plane wave of amplitude A and frequency ω propagating in the direction of the vector $\vec{k}$, which in turn lies on a line drawn from the origin to the point (4, 2, 1). *Hint:* First determine $\vec{k}$ and then dot it with $\vec{r}$.

2.43* Write an expression in Cartesian coordinates for a harmonic plane wave of amplitude A and frequency ω propagating in the positive x-direction.

2.44 Show that $\psi(\vec{k}\cdot\vec{r}, t)$ may represent a plane wave where $\vec{k}$ is normal to the wavefront. *Hint:* Let $\vec{r}_1$ and $\vec{r}_2$ be position vectors drawn to any two points on the plane and show that $\psi(\vec{r}_1, t) = \psi(\vec{r}_2, t)$.

2.45* Make up a table with columns headed by values of θ running from $-\pi/2$ to 2π in intervals of $\pi/4$. In each column place the corresponding value of $\sin\theta$, and beneath those the values of $2\sin\theta$. Next add these, column by column, to yield the corresponding values of the function $\sin\theta + 2\sin\theta$. Plot each of these three functions, noting their relative amplitudes and phases.

2.46* Make up a table with columns headed by values of θ running from $-\pi/2$ to 2π in intervals of $\pi/4$. In each column place the corresponding value of $\sin\theta$, and beneath those the values of $\sin(\theta - \pi/2)$. Next add these, column by column, to yield the corresponding values of the function $\sin\theta + \sin(\theta - \pi/2)$. Plot each of these three functions, noting their relative amplitudes and phases.

2.47* With the last two problems in mind, draw a plot of the three functions (a) $\sin\theta$, (b) $\sin(\theta - 3\pi/4)$, and (c) $\sin\theta + \sin(\theta - 3\pi/4)$. Compare the amplitude of the combined function (c) in this case with that of the previous problem.

2.48* Make up a table with columns headed by values of kx running from $x = -\lambda/2$ to $x = +\lambda$ in intervals of x of $\lambda/4$. In each column place the corresponding values of $\cos kx$ and beneath that the values of $\cos(kx + \pi)$. Next plot the three functions $\cos kx$, $\cos(kx + \pi)$, and $\cos kx + \cos(kx + \pi)$.

PROBLEMS

Complete solutions to all problems—except those with an asterisk—can be found in the back of the book.

3.1 Consider the plane electromagnetic wave (in SI units) given by the expressions $E_x = 0$, $E_y = 2\cos[2\pi \times 10^{14}(t - x/c) + \pi/2]$, and $E_z = 0$.

(a) What are the frequency, wavelength, direction of motion, amplitude, initial phase angle, and polarization of the wave?

(b) Write an expression for the magnetic flux density.

3.2 Write an expression for the $\vec{\mathbf{E}}$- and $\vec{\mathbf{B}}$-fields that constitute a plane harmonic wave traveling in the $+z$-direction. The wave is linearly polarized with its plane of vibration at 45° to the yz-plane.

3.3* Considering Eq. (3.30), show that the expression

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega \vec{\mathbf{B}}$$

is correct as it applies to a plane wave for which the direction of the electric field is constant.

3.4* Imagine an electromagnetic wave with its $\vec{\mathbf{E}}$-field in the y-direction. Show that Eq. (3.27)

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

applied to the harmonic wave $\vec{\mathbf{B}}$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 \cos(kx - \omega t) \qquad \vec{\mathbf{B}} = \vec{\mathbf{B}}_0 \cos(kx - \omega t)$$

yields the fact that

$$E_0 = cB_0$$

in agreement with Eq. (3.30).

3.5* An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{\mathbf{E}} = (-6\hat{\mathbf{i}} + 3\sqrt{5}\hat{\mathbf{j}})(10^4\ \mathrm{V/m})e^{i[\frac{1}{3}(\sqrt{5}x + 2y)\pi \times 10^7 - 9.42 \times 10^{15} t]}$$

Find (a) the direction along which the electric field oscillates, (b) the scalar value of amplitude of the electric field, (c) the direction of propagation of the wave, (d) the propagation number and wavelength, (e) the frequency and angular frequency, and (f) the speed.

3.6 The electric field of an electromagnetic wave traveling in the positive x-direction is given by

$$\vec{\mathbf{E}} = E_0 \hat{\mathbf{j}} \sin\frac{\pi z}{z_0} \cos(kx - \omega t)$$

(a) Describe the field verbally. (b) Determine an expression for k. (c) Find the phase speed of the wave.

3.7* A 550-nm harmonic EM-wave whose electric field is in the z-direction is traveling in the y-direction in vacuum. (a) What is the frequency of the wave? (b) Determine both ω and k for this wave. (c) If the electric field amplitude is 600 V/m, what is the amplitude of the magnetic field? (d) Write an expression for both $E(t)$ and $B(t)$ given that each is zero at $x = 0$ and $t = 0$. Put in all the appropriate units.

3.8* Calculate the energy input necessary to charge a parallel plate capacitor by carrying charge from one plate to the other. Assume the energy is stored in the field between the plates and compute the energy per unit volume, u_E, of that region, that is, Eq. (3.31). *Hint:* since the electric field increases throughout the process, either integrate or use its average value $E/2$.

3.9* Starting with Eq. (3.32), prove that the energy densities of the electric and magnet fields are equal ($u_E = u_B$) for an electromagnetic wave.

3.10 The time average of some function $f(t)$ taken over an interval T is given by

$$\langle f(t)\rangle_T = \frac{1}{T}\int_t^{t+T} f(t')\,dt'$$

where t' is just a dummy variable. If $\tau = 2\pi/\omega$ is the period of a harmonic function, show that $\vec{\mathbf{r}}$

$$\langle \sin^2(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\rangle = \tfrac{1}{2}$$

$$\langle \cos^2(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\rangle = \tfrac{1}{2}$$

and

$$\langle \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)\rangle = 0$$

when $T = \tau$ and when $T >> \tau$.

3.11* Show that a more general formulation of the previous problem yields

$$\langle \cos^2 \omega t\rangle_T = \tfrac{1}{2}[1 + \operatorname{sinc} \omega T \cos 2\omega t]$$

for any interval T.

3.12* With the previous problem in mind, prove that

$$\langle \sin^2 \omega t\rangle_T = \tfrac{1}{2}[1 - \operatorname{sinc} \omega T \cos 2\omega t]$$

for any interval T.

3.13* Prove that the irradiance of a harmonic EM-wave is given by

$$I = \frac{1}{2c\mu_0} E_0^2$$

and then determine the average rate at which energy is transported per unit area by a plane wave having an amplitude of 15.0 V/m.

3.14* A light bulb puts out 20 W of radiant energy (most of it IR). Assume it to be a point source and calculate the irradiance 1.00 m away.

3.15* Consider a linearly polarized plane electromagnetic wave traveling in the $+x$-direction in free space having as its plane of vibration the xy-plane. Given that its frequency is 10 MHz and its amplitude is $E_0 = 0.08$ V/m,

(a) Find the period and wavelength of the wave.

(b) Write an expression for $E(t)$ and $B(t)$.

(c) Find the flux density, $\langle S\rangle$, of the wave.

3.16* On average, the net electromagnetic power radiated by the Sun, its so-called *luminosity* (L), is 3.9×10^{26} W. Determine the mean amplitude of the electric field due to all the radiant energy arriving at the top of Earth's atmosphere (1.5×10^{11} m from the Sun).

3.17 A linearly polarized harmonic plane wave with a scalar amplitude of 10 V/m is propagating along a line in the xy-plane at 45° to the x-axis with the xy-plane as its plane of vibration. Please write a vector expression describing the wave assuming both k_x and k_y are positive. Calculate the flux density taking the wave to be in vacuum.

3.18 Pulses of UV lasting 2.00 ns each are emitted from a laser that has a beam of diameter 2.5 mm. Given that each burst carries an energy of 6.0 J, (a) determine the length in space of each wavetrain, and (b) find the average energy per unit volume for such a pulse.

3.19* A laser provides pulses of EM-radiation in vacuum lasting 10^{-12} s. If the radiant flux density is 10^{20} W/m^2, determine the amplitude of the electric field of the beam.

3.20 A 1.0-mW laser has a beam diameter of 2 mm. Assuming the divergence of the beam to be negligible, compute its energy density in the vicinity of the laser.

3.21* A cloud of locusts having a density of 100 insects per cubic meter is flying north at a rate of 6 m/min. What is the flux density of locusts? That is, how many cross an area of 1 m^2 perpendicular to their flight path per second?

3.22 Imagine that you are standing in the path of an antenna that is radiating plane waves of frequency 100 MHz and flux density 19.88 $\times 10^{-2}$ W/m^2. Compute the photon flux density, that is, the number of photons per unit time per unit area. How many photons, on the average, will be found in a cubic meter of this region?

3.23* How many photons per second are emitted from a 100-W yellow lightbulb if we assume negligible thermal losses and a quasi-monochromatic wavelength of 550 nm? In actuality only about 2.5% of the total dissipated power emerges as visible radiation in an ordinary 100-W lamp.

3.24 A 3.0-V flashlight bulb draws 0.25 A, converting about 1.0% of the dissipated power into light ($\lambda \approx 550$ nm). If the beam has a cross-sectional area of 10 cm^2 and is approximately cylindrical,

(a) How many photons are emitted per second?

(b) How many photons occupy each meter of the beam?

(c) What is the flux density of the beam as it leaves the flashlight?

3.25* An isotropic quasimonochromatic point source radiates at a rate of 100 W. What is the flux density at a distance of 1 m? What are the amplitudes of the $\vec{\mathbf{E}}$- and $\vec{\mathbf{B}}$-fields at that point?

3.26 Using energy arguments, show that the amplitude of a cylindrical wave must vary inversely with $\sqrt{r}$. Draw a diagram indicating what's happening.

3.27* What is the momentum of a 10^{19}-Hz X-ray photon?

3.28 Consider an electromagnetic wave impinging on an electron. It is easy to show kinematically that the average value of the time rate-of-change of the electron's momentum $\vec{\mathbf{p}}$ is proportional to the average value of the time rate-of-change of the work, W, done on it by the wave. In particular,

$$\left\langle\frac{d\vec{\mathbf{p}}}{dt}\right\rangle = \frac{1}{c}\left\langle\frac{dW}{dt}\right\rangle\hat{\mathbf{i}}$$

Accordingly, if this momentum change is imparted to some completely absorbing material, show that the pressure is given by Eq. (3.51).

3.29* Derive an expression for the radiation pressure when the normally incident beam of light is totally reflected. Generalize this result to the case of oblique incidence at an angle θ with the normal.

3.30 A completely absorbing screen receives 300 W of light for 100 s. Compute the total linear momentum transferred to the screen.

3.31 The average magnitude of the Poynting vector for sunlight arriving at the top of Earth's atmosphere (1.5×10^{11} m from the Sun) is about 1.4 kW/m^2.

(a) Compute the average radiation pressure exerted on a metal reflector facing the Sun.

(b) Approximate the average radiation pressure at the surface of the Sun whose diameter is 1.4×10^9 m.

3.32* A surface is placed perpendicular to a beam of light of constant irradiance (I). Suppose that the fraction of the irradiance absorbed by the surface is α. Show that the pressure on the surface is given by

$$\mathcal{P} = (2 - \alpha)I/c$$

3.33* A light beam with an irradiance of 2.00×10^6 W/m^2 impinges normally on a surface that reflects 70.0% and absorbs 30.0%. Compute the resulting radiation pressure on the surface.

3.34 What force on the average will be exerted on the (40 m $\times$ 50 m) flat, highly reflecting side of a space station wall if it's facing the Sun while orbiting Earth?

3.35 A parabolic radar antenna with a 2-m diameter transmits 200-kW pulses of energy. If its repetition rate is 500 pulses per second, each lasting 2 μs, determine the average reaction force on the antenna.

3.36 Consider the plight of an astronaut floating in free space with only a 10-W lantern (inexhaustibly supplied with power). How long will it take to reach a speed of 10 m/s using the radiation as propulsion? The astronaut's total mass is 100 kg.

3.37 Consider the uniformly moving charge depicted in Fig. 3.26*b*. Draw a sphere surrounding it and show via the Poynting vector that the charge does not radiate.

3.38* A plane, harmonic, linearly polarized light wave has an electric field intensity given by

$$E_z = E_0 \cos \pi\, 10^{15}\left(t - \frac{x}{0.65c}\right)$$

while traveling in a piece of glass. Find

(a) The frequency of the light.

(b) Its wavelength.

(c) The index of refraction of the glass.

3.39* What is the speed of light in diamond if the index of refraction is 2.42?

3.40* Given that the wavelength of a lightwave in vacuum is 540 nm, what will it be in water, where $n = 1.33$?

3.41* Determine the index of refraction of a medium if it is to reduce the speed of light by 10% as compared to its speed in vacuum?

3.42 If the speed of light (the phase speed) in Fabulite ($SrTiO_3$) is 1.245×10^8 m/s, what is its index of refraction?

3.43* What is the distance that yellow light travels in water (where $n = 1.33$) in 1.00 s?

3.44* A 500-nm lightwave in vacuum enters a glass plate of index 1.60 and propagates perpendicularly across it. How many waves span the glass if it's 1.00 cm thick?

3.45* Yellow light from a sodium lamp ($\lambda_0 = 589$ nm) traverses a tank of glycerin (of index 1.47), which is 20.0 m long, in a time t_1. If it takes a time t_2 for the light to pass through the same tank when filled with carbon disulfide (of index 1.63), determine the value of $t_2 - t_1$.

3.46* A lightwave travels from point A to point B in vacuum. Suppose we introduce into its path a flat glass plate ($n_g = 1.50$) of thickness $L = 1.00$ mm. If the vacuum wavelength is 500 nm, how many waves span the space from A to B with and without the glass in place? What phase shift is introduced with the insertion of the plate?

3.47 The low-frequency relative permittivity of water varies from 88.00 at 0°C to 55.33 at 100°C. Explain this behavior. Over the same range in temperature, the index of refraction ($\lambda = 589.3$ nm) goes from roughly 1.33 to 1.32. Why is the change in n so much smaller than the corresponding change in K_E?

3.48 Show that for substances of low density, such as gases, which have a single resonant frequency ω_0, the index of refraction is given by

$$n \approx 1 + \frac{Nq_e^2}{2\epsilon_0 m_e(\omega_0^2 - \omega^2)}$$

3.49* In the next chapter, Eq. (4.47), we'll see that a substance reflects radiant energy appreciably when its index differs most from the medium in which it is embedded.

(a) The dielectric constant of ice measured at microwave frequencies is roughly 1, whereas that for water is about 80 times greater—why?

(b) How is it that a radar beam easily passes through ice but is considerably reflected when encountering a dense rain?

3.50 Fuchsin is a strong (aniline) dye, which in solution with alcohol has a deep red color. It appears red because it absorbs the green component of the spectrum. (As you might expect, the surfaces of crystals of fuchsin reflect green light rather strongly.) Imagine that you have a thin-walled hollow prism filled with this solution. What will the spectrum look like for incident white light? By the way, anomalous dispersion was first observed in about 1840 by Fox Talbot, and the effect was christened in 1862 by Le Roux. His work was promptly forgotten, only to be rediscovered eight years later by C. Christiansen.

3.51* Take Eq. (3.71) and check out the units to make sure that they agree on both sides.

3.52 The resonant frequency of lead glass is in the UV fairly near the visible, whereas that for fused silica is far into the UV. Use the dispersion equation to make a rough sketch of n versus ω for the visible region of the spectrum.

3.53* Show that Eq. (3.70) can be rewritten as

$$(n^2 - 1)^{-1} = -C\lambda^{-2} + C\lambda_0^{-2}$$

where $C = 4\pi^2 c^2 \epsilon_0 m_e / Nq_e^2$.

3.54 Augustin Louis Cauchy (1789–1857) determined an empirical equation for $n(\lambda)$ for substances that are transparent in the visible. His expression corresponded to the power series relation

$$n = C_1 + C_2/\lambda^2 + C_3/\lambda^4 + \cdots$$

where the Cs are all constants. In light of Fig. 3.41, what is the physical significance of C_1?

3.55 Referring to the previous problem, realize that there is a region between each pair of absorption bands for which the Cauchy Equation (with a new set of constants) works fairly well. Examine Fig. 3.41: what can you say about the various values of C_1 as ω decreases across the spectrum? Dropping all but the first two terms, use Fig. 3.40 to determine approximate values for C_1 and C_2 for borosilicate crown glass in the visible.

3.56* Crystal quartz has refractive indexes of 1.557 and 1.547 at wavelengths of 410.0 nm and 550.0 nm, respectively. Using only the first two terms in Cauchy's Equation, calculate C_1 and C_2 and determine the index of refraction of quartz at 610.0 nm.

3.57* In 1871 Sellmeier derived the equation

$$n^2 = 1 + \sum_j \frac{A_j\lambda^2}{\lambda^2 - \lambda_{0j}^2}$$

where the A_j terms are constants and each λ_{0j} is the vacuum wavelength associated with a natural frequence ν_{0j}, such that $\lambda_{0j}\nu_{0j} = c$. This formulation is a considerable practical improvement over the Cauchy Equation. Show that where $\lambda >> \lambda_{0j}$, Cauchy's Equation is an approximation of Sellmeier's. *Hint:* Write the above expression with only the first term in the sum; expand it by the binomial theorem; take the square root of n^2 and expand again.

3.58* If an ultraviolet photon is to dissociate the oxygen and carbon atoms in the carbon monoxide molecule, it must provide 11 eV of energy. What is the minimum frequency of the appropriate radiation?