**Homework assignments are due at 11 pm (building close).** Problems worth 4 pts each except when noted

Labs are shown in red

H1. P0.1,3,6,14,18,20  Hint: on 18 use eq 0.30. If you need **more examples** with complex numbers, google "complex numbers tutorial". For more examples on vector calculus, google "curl and divergence".

H2. P1. 5 and problems 2x,2y below.

**Prob 2x.** If you want to create an E-field \( \vec{E} = (7x^2y^3 \hat{x} + 2z^4 \hat{y}) \cos \omega t \) from a changing charge distribution and currents in wires, find:

a. \( \rho(x,y,z,t) \) that is required

b. \( \frac{\partial \vec{B}(x,y,z,t)}{\partial t} \) that must be present

c. Find some B and J that are consistent with this and show that E is a solution to the wave equation eqn 1.40. [Note: when you find B and J consistent with all of Maxwell's equations, then E will always be a solution to the full wave equation 1.40. So in showing it obeys the wave equation, you're showing that the wave equation satisfies Maxwell's equations in this particular case].

d. Some of J will come from changing \( \rho \), and some of J you will have to supply from current in neutral wires. Find the terms in J(x,y,z) you would have to supply in wires (the divergence free terms).

**Prob 2y.** The gray dot represents an infinite straight wire of radius R that has current \( I_o \) coming out of the page. We call this direction \( \hat{z} \). Inside the wire there is uniform current density \( J = \frac{I_o}{\pi R^2} \) (current/area).

a. Using a right hand rule from Phys 220, show that B is in the direction of \( \hat{\phi} \) (using cylindrical coordinates \( r, \phi, z \)). In other words \( \vec{B} = B(r)\hat{\phi} \). Find the strength of the B-field for \( r < R \) and \( r > R \), in terms of \( I_o \). See the hints on P1.3

b. In cylindrical coordinates the curl is written:

\[
\nabla \times \vec{u} = \hat{\rho} \left( \frac{1}{r} \frac{\partial u_z}{\partial \phi} - \frac{\partial u_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) + \hat{z} \frac{1}{r} \left[ \frac{\partial (ru_\phi)}{\partial r} - \frac{\partial u_r}{\partial \phi} \right]
\]
Using this and the B’s you found in a), show that $\nabla \times \vec{B} = \mu_0 \vec{J}$ is true.

c. Now we make J increase with time, so that B increases with time: $J(t) = \frac{I_o t}{\pi R^2 t_o}$. $t_o$ is a constant.

From $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, show that there is an E-field induced along the z axis, and find its magnitude for $r < R$.

H3. P1.6, 9, L1.10 (8 pts), and Probs 3x,3y below.

**Prob3x.** a) An electromagnetic wave is traveling in the (2, -1, 3) direction, and its E-field oscillates in the (1, 2, 0) direction. Note that neither of those vectors has been normalized. The wave’s electric field amplitude is 5 V/m, its wavelength is 13 m, and its speed is obviously $3 \times 10^8$ m/s. Write down a complex exponential plane wave which describes the electric field of the wave. b) We can specify the wave so that $|\vec{E}(t = 0)|$ has any amplitude from 0 to the full amplitude $A$. Add a phase shift so that at $t = 0$ and $\vec{r} = 0$ (or any fixed $\vec{r}$) the real part of E (which we take to be the physical part) is $\frac{1}{2}$ its maximum.

**Prob3y.** a) Starting from equ 2.13 $\nabla^2 \vec{E} - \varepsilon_o \mu_o \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_o \frac{\partial \vec{J}_p}{\partial t} = \mu_o \frac{\partial^2 \vec{P}}{\partial t^2}$, using complex plane waves and $\vec{P} = \varepsilon_o \chi \vec{E}$, learn by heart the derivation (shown in class and text) of how $k$ depends on the material: equ 2.17 $k = \frac{\omega}{c} \sqrt{1 + \frac{\chi}{\varepsilon}}$. Practice until you can turn in a derivation you have done from beginning to end without any help from text, notes, or others.

b) Learn by heart the relations below that come from the derivation above and the definition of $k$, and previous physics. When you can write them by memory, (use the blank set to the right to practice as you cover the answers), then write a set to turn in.

When absorption is negligible,

\[
\begin{align*}
    n &= \sqrt{1 + \frac{\chi}{\varepsilon}} \\
    k &= \frac{2\pi}{\lambda} = \frac{n2\pi}{\lambda_{\text{vac}}} = n \frac{\omega}{c} \\
    \nu &= \frac{c}{n} = \frac{f\lambda}{\nu} = \frac{\omega}{k} \\
    \omega &= 2\pi f \\
    \lambda &= \frac{\lambda_{\text{vac}}}{n} \\
    \end{align*}
\]
H4. P2.3 (8 pts) 5,7 and Prob 4.x. On 2.3 use Mathematica and plot \( n(\omega), \kappa(\omega) \). The amplitude of the charge displacement will be much less than an angstrom. You know you’re doing it right if most of the values of \( n \) and \( \kappa \) at the end are between 0.1 and 2.

**Prob 4.x.** a) Starting from \( \chi(\omega) = \frac{q_e^2 N}{\varepsilon m_e \omega^3 - \omega - i \omega \gamma} \) which is for a dielectric (insulator), learn by heart the few (2 or 3) steps and arguments we used in class to get \( (n + i\kappa)^2 = 1 - \frac{q_e^2 N}{\varepsilon m_e \omega^3} = 1 - \frac{\omega_p^2}{\omega^2} \) for a metal with small damping. Submit your steps (with their arguments) when you’ve learned them.

b) Argue from \( (n + i\kappa)^2 = 1 - \frac{\omega_p^2}{\omega^2} \) that one of \( n, \kappa \) must be zero below \( \omega_p \), and the other must be zero above it. Explain how a purely real index vs a purely imaginary index correspond to the cases of good transmission vs poor transmission (in the case of metals, the poor transmission shows up as reflection, not absorption).

H5. P2.10, 3.2,5 and Prob 5x,y below. On 3.2 just do it for the first two rows of equations, not all 4.

**Prob 5x.** Extending P2.10, assuming it’s a green laser of \( \lambda = 500 \) nm which fires 100 times per second, find d) the number of photons in each pulse e) the average power of the laser (averaged over long times), f) the average number of photons per second it emits, g) the photon density (number/m^3 and number/angstrom^3) in the region where the beam is focused to 5 mm (use the E-field from b) .

**Background for Problem 5x**

The idea you need is that if there is some energy in a beam, pulse or field, it can be expressed by a number of photons: \( \text{energy} = N_{\text{photons}} \hbar \omega \). So you can relate this idea to the power and intensity of a laser beam. To relate it to the field strength itself, you have to write the intensity or energy density in terms of both the field and the photon energy. For example \( u_{\text{field}} = \frac{\varepsilon_0}{2} |E|^2 \), but it must also be true that \( u = \frac{N_{\text{photons}} \hbar \omega}{V} \), where \( \frac{N_{\text{photons}}}{V} \) is the photon density (number/volume).

**Prob 5y.** Given fig 3.1 and having memorized the boundary condition that E components parallel to the boundary must be equal on both sides, derive the fact of frequency conservation, the law of reflection and Snell’s law, i.e. eqns 3.4 through 3.7. Include the necessary arguments. Turn it in after you can do it by heart.

H6. L3.4 (8 pts), P3.12,13, and Probs 6x,y. (On P3.12 and P6x, the physics we need is in the y and z dependence of eq 3.44). On L3.4 measure and plot reflectance vs angle for s- and p-polarizations reflecting from a metal. Don’t expect it to match prob 3.13, as it’s probably aluminum, not silver.

**Prob 6x:** extending P3.12

b) find the wavelength (i.e. repeat distance) of the evanescent wave that propagates along the surface (see hint below).

c) on Mathematica, plot the decay length (the length you found in the first part of 3.12) vs angle in the critical region. d) plot \(|r|\) and the phase of \( r \) vs angle in the critical region for both s and p polarization. Note, on 3.12 and 13, for both just use Fresnel’s coefficients eq 3.20-23 directly in Mathematica, which handles complex numbers fine...you don’t need eqn 3.49,50.
Hint on b): two ways to do it... draw the geometry of Fig 3.9 and do some trig, or get it from this idea: when we see a wave of the form $\exp(i(au + bv + cw - \omega t))$, where $u, v, w$ are coordinates, we know that the factor that multiplies any coordinate must equal $\frac{2\pi}{d}$, where $d$ is the distance between crests in the direction of that coordinate. For example $a = \frac{2\pi}{d_u}$ because when $u$ increases by $d_u = \frac{2\pi}{a}$ (and the other variables don’t change), then the wave has picked up a factor of $\exp(i2\pi) = 1$, hence it’s gone through a full cycle.

**Prob 6y:** Turn these in after you can do them by heart

a) Knowing that the transmitted ray goes to $\pi/2$ at the critical angle, derive the critical incident angle from Snell’s law.  
b) For incident angles above critical, derive the decay length (distance to decrease by factor of $e$) of the evanescent wave with this strategy: If $z$ is the normal to the interface, show how Snell’s law determines an imaginary $\cos\theta$ that is part of $k_z$.  Your results should agree with the $z$ dependence in eq 3.44.

H7. P4.1,2,3,4,5.  On 2, add: b) (4 pts also) give an explanation with Phys 123 concepts and skills (phase shifts and thicknesses) why the maxima and minima occur at the ones you see.  It's easiest to explain where $R$ is large and small, which corresponds to where $T$ is small and large.

H8. P4. 6,8,10, L4.7.  Simplify lab 4:y: a) You don’t have to plot anything, just take a few ratios of intensities to convince the grader that $T$ decays slowly for $d$ around $\lambda$, then exponentially for $d >> \lambda$.  
b) Suppose you remove the 2<sup>nd</sup> prism, and could accurately measure the decaying evanescent wave amplitude vs distance $z$ from the prism.  Does the theory for TIR give you the same behavior as frustrated TIR ($T$ decays slowly for $z$ around $\lambda$, then exponentially for $z >> \lambda$)? Explain.

H9.  P4.15, 16, 17.  On 17, you can just try all the possibilities; I wrote a simple double loop to test all the combinations of materials to find the lowest $R$.

H10. P5.2, phase matching problem R40 on pg 222, and 10x below. On R40, the physics is that all the n's grow with frequency, so an index "ellipse" at one frequency might intersect an index "sphere" at another frequency.  You can do this with polar plots in $\theta$, but maybe the easiest way is to plot vs $\theta$: normal plots n-e($\omega, \theta$), n-e($2\omega, \theta$), n-o($\omega$), n-o($2\omega$).  Of course the n-o's don't depend on $\theta$, and will be straight lines.  Look for intersections of an n-e at one frequency with an n-o at another frequency.

**Prob 10x.** (8pts) Light from air enters a uniaxial crystal with the optical axis along the $z$ axis shown.  The $y$ direction is into the page.  The optic axis is 45 degrees to the crystal surface.  The incident $k$-vector is in red: 

$$ \vec{k}_{inc} = \frac{k}{\sqrt{5}} (2\hat{x} - \hat{z}) $$

The $E$-field (double arrows) is in the plane of the figure with direction $\vec{E}_{inc} = \frac{E_o}{\sqrt{5}} (\hat{x} + 2\hat{z})$.  The optical properties of the crystal are given by $\chi_x = 3, \chi_y = 3, \chi_z = 8$.

a) Find the numerical values for the two constants $n_o, n_e$, and plot the function $n_e(\theta_{k-\omega})$ as vs. angle between the OA and the
unknown \( k_i \) in the crystal. Also plot \( n_e(\theta_i) \) where \( \theta_i \) is measured from the crystal normal.

b) Find \( \theta_i \) in degrees from the crystal normal.

c) Find the angle \( \theta_i \) from Snell’s law (measured from the crystal normal). i.e. find where \( \sin \theta_i \) (a constant) and \( n_e(\theta_i) \sin \theta_i \) are equal. What is \( \theta_i \) from the OA? Check: I got 52 deg from OA axis.

d) Knowing \( \theta_i \), write a unit vector for \( \vec{k}_i \) (in the x,z coordinate system).

e) From \( \vec{k}_i \cdot (\epsilon_n \vec{E}_i + \vec{P}) = 0 \) find the ratios of the components of \( E \) in the crystal, and hence the direction of \( \vec{E}_i \) in the crystal coordinate system. Find the angle between \( E \) and \( \vec{k}_i \)

f) Knowing that \( S \) and \( E \) are perpendicular, find the angle that \( S \) makes with the optic axis. Make a sketch showing \( E \), \( K \) and \( S \) in the crystal. Check: I got \( S \) is 34 degrees from the OA

H11. P6. 2,4,5,6,8, and 11.x below.

Prob 11.x. An initial light state is described by \( \begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix} \). \( x \) is the horizontal direction, \( y \) vertical.

a) Describe the initial state of the light in as much detail as possible.

b) It strikes a quarter-waveplate with its fast along the \( y \) axis, and then a linear polarizer that transmits at -45 degrees from the +x axis. What fraction of the initial intensity is transmitted? Use Jones matrices.

c) Just before the final polarizer, describe the state of the light in as much detail as possible.

H12. P7.1,3,4 and 12x(12 pts),y below.

Prob 12x: Complete the polarizer activity. In class you will borrow three linear polarizers. You can do this alone, or in pairs if you do all the activities together.

Prob 12y: Suppose in some region the index is approximated by \( n(\lambda) = n_o + b\lambda^3 \).

a) Find the phase velocity of a wave with wavelength \( \lambda \) in terms of these symbols.

b) Find the group velocity of a wave with average wavelength \( \bar{\lambda} \). A couple of ways to do this, but you might use: \( \begin{bmatrix} \frac{\partial k}{\partial \omega} \\ \frac{\partial k}{\partial \bar{\lambda}} \end{bmatrix} = \begin{bmatrix} \frac{\partial k}{\partial \lambda} \\ \frac{\partial \lambda}{\partial \omega} \end{bmatrix} \) Ans: \( v_g = \frac{c}{n_o - 2b\lambda^3} \)

H13. P0.21,23,24 (Fourier theory), P7.5
14x. Continues 28 above. a) Find the Fourier transform of a \( f(t) \) consisting of the sum of three identical Gaussian optical pulses of the form \( e^{-t^2/2T^2} \sin \omega_o t \), but separated by a time \( t_i \). The three Gaussians are centered at \( t=0 \), \( t=t_i \), and \( t=-t_i \).

Use a convolution theorem. The three-pulse function \( f(t) \) is a convolution of \( e^{-t^2/2T^2} \sin \omega_o t \) with three delta functions. You will use the results of 28 above. Here is an easy check: when \( t_i=0 \), the three pulses are on top of each other, so you should get 3x the answer in 28.

b) For \( \omega_o = 1 \), \( T = 8 \), \( t_i = 30 \), plot \( f(t) \). Plot the imaginary part of \( f(\omega) \). Comment on how your plot (and the plot in 28d) illustrate the concepts we’ve learned about Fourier transforms of pulses, and how the two are different.

H15. P7.7,P8.2,3,4,5, and Prob 15x:

Note 1 In 8.2 the hint says “use \( \Delta\omega \approx \frac{2\pi c}{\lambda_o} |\Delta\lambda| \).” Every physicist must know how to derive this simple relationship by differentiating \( \omega = ck = \frac{2\pi c}{\lambda} \). Never use \( \Delta\omega \approx \frac{2\pi c}{|\Delta\lambda|} \) …can you see why it’s wrong?

Note 2: on 8.4, for the plotting of \( I(\tau) \), you can set \( \omega_o = 1 \), and use \( \Delta\omega = 0.1 \).

Note 3: on 8.5, for the plotting you can set \( \omega_o = 1 \), and use \( \Delta\omega = 0.1 \).

Prob 15x: Continue 8.4 add: a) Find \( t_c \) from eq. 8.13. Relate it to your plot and discuss whether it obeys the uncertainty principle discussed in class. b) From eq. 8.16, find the fringe visibility at \( \tau = t_c \) and \( 2t_c \), and relate it to your plot.

H15. P7.7,P8.2,3,4,5, and Prob 15x:

Note 1 In 8.2 the hint says “use \( \Delta\omega \approx \frac{2\pi c}{\lambda_o} |\Delta\lambda| \).” Every physicist must know how to derive this simple relationship by differentiating \( \omega = ck = \frac{2\pi c}{\lambda} \). Never use \( \Delta\omega \approx \frac{2\pi c}{|\Delta\lambda|} \) …can you see why it’s wrong?

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Prob 15x: Continue 8.4 add: a) Find \( t_c \) from eq. 8.13. Relate it to your plot and discuss whether it obeys the uncertainty principle discussed in class. b) From eq. 8.16, find the fringe visibility at \( \tau = t_c \) and \( 2t_c \), and relate it to your plot.

H16. 8.9a, and 16x,y. Note on 8.9a, and 16y if you change everything about the source to angles (\( \theta_{\text{max}} = \frac{a}{2R} \)) before you try to integrate, it should be easier. See the last slide of the lecture.

16x: Coherence of starlight

The nearest star (other than our sun) to us is Proxima Centauri at a distance of 30 trillion kilometers, and it has an angular diameter of 2 millionth of a degree or 7 milliarseconds (1 milliarcsecond is 1 thousandth of an arcsecond which is one sixtieth of an arcminute which is one sixtieth of a degree…thanks to the Babylonians, who loved multiples of 60). The only reason we know its diameter is because of interferometry.

a. If an astronomer filters the white starlight through a green filter, \( \lambda = 500 \) nm, \( \Delta\lambda = 50 \) nm, what are the approximate temporal coherence length and coherence time of the light after passing through the filter? Use the uncertainty principle.
b. Using concepts from class, sketch the power spectrum $I(\omega)$ for this light, and label the approximate width in rad/sec, as well as the position of the peak in rad/sec.

c. Using concepts from class, sketch the interferogram $I(\tau)$ for delay $\tau$ for this single star. Label the approximate width of the wiggles in femtoseconds (fs), as well as the approximate coherence time.

d. What is the approximate spatial coherence length of this light due to the diameter of the star? Use the simple estimate $h_c = \frac{\lambda}{\Delta \theta}$. Use a single $\lambda = 500$ nm for this analysis. This length is approximately the baseline of a stellar interferometer needed to resolve the star’s diameter.

16y: In 8.9a) Instead of a line source with constant intensity, we consider a 2-D disc with constant intensity like the sun. We can still do a one-dimensional problem, if we use for $y' < \frac{a}{2}$, $I(y'\prime) = I_0 \sqrt{1-\frac{4y'^{2}}{a^2}}$, in other words, the intensity is proportional to the width of the disc as we move along $y$, so the effective 1-D intensity is more concentrated near $y=0$ than for a line source. For $y' > \frac{a}{2}$, $I = 0$.

i) Find $\gamma(h)$ in terms of special functions...do it it Mathematica.

ii) Using $\theta_{\text{max}} = \frac{a}{2R}$, plot Re($\gamma(h)$) vs $h$ (in units of $\lambda$), and also plot Re($\gamma(h)$) for the line source in a). Do this for two cases: $\theta_{\text{max}} = 0.01$ rad and 0.0001 rad. Which has a longer spatial coherence length $h_c$, the line or disc source? Why does that make sense?

H17. P9.3,4 and 17x:
17x: The text’s version of the eikonal equation $(\nabla R(\vec{r}) = n(\vec{r}) \hat{s}(\vec{r})$) is usually written in another form: we eliminate $R$ by differentiating, and get $\frac{d}{ds}(n\hat{s}) = \nabla n$. This says that the gradient of $n$ determines how the direction $\hat{s}$ of the ray changes as you move a distance $ds$ along the path. This form can be used in a computer solution to trace any ray’s path. Let’s restrict $n$ so it changes in only one direction, for example $y$. Then we can write $\nabla n = n' \hat{y}$. If the direction of the ray in the x-y plane is given by $\theta$, the direction is the unit vector $\hat{s} = \cos \theta \hat{x} + \sin \theta \hat{y}$.

a) show that $\frac{d}{ds}(n(y)\hat{s}) = \nabla n(y)$ becomes $d\theta = \frac{1}{n} n' \cos \theta ds$. $s$ is the variable that tells you where you are along the ray path.

Hint: $\frac{d}{ds} = \frac{d\theta}{ds} \frac{d}{d\theta}$. After differentiating, dot both sides with some unit vector that will give you a scalar equation in terms of $d\theta$.

b) Argue (including a sketch of how a couple of rays change direction ) that $d\theta = \frac{1}{n} n' \cos \theta ds$ means if the gradient $n'$ doesn’t change sign, then rays not aligned with the gradient will eventually end up being aligned with the gradient ($y$ axis), if they travel long enough.
H18. P9.7,9,12,13

H19. P9. 15,17

H20. L14, 20.x,y,z below

20.x  Thin lens addition formula. Using either the ABCD_total method or the principal planes method, join two (or any number of) thin lenses together without any space between them, and show that \( 1/f_{\text{total}} = 1/f_1 + 1/f_2 \). Note: this is why opticians use "optical power" = \( 1/f \) for a lens (unit 1/m called a "diopter"), because they can add the "powers" very easily from the many test lenses to get the final prescription.

20.y  Spherical wave addition. Our diffraction analysis methods all rely on Huygen’s principle of adding E-fields from spherical waves at every point in an aperture. Here we will consider just three equal strength point sources at \( y' = a, 0 \) and \( -a \). A screen is placed at \( z = d \). Each source emits at the same phase with wavelength \( \lambda \).

\[
\begin{array}{c}
\text{+a} \\
0 \\
\text{-a}
\end{array} \quad (y,d) \\
\quad z-d
\]

a) Scalar addition approximation: Write the sum \( E(y,d) \) if each source emits a scalar wave \( A e^{ikR} \). Plot the shape of \( I(y,d) \propto E \ast E \) if \( a = 100\lambda \), for \( d = a \) and \( d = 1.5a \).

b) Fresnel (near field) regime. You need \( d>>a \) so that the angles to the screen are small. Keep doubling \( d \) and look at the patterns. As long as the shapes keep changing (not just expanding), this is the Fresnel regime. Plot two of these that look pretty different and give the \( d \)'s.

c) Fraunhofer (far field) regime. Keep doubling \( d \) until you find a case where the shape of the pattern doesn't change (just expands with \( d \), so it's the same angular form). Plot two of these that look similar for very different \( d \)'s and give an approximate \( d \) where you see the Fraunhofer regime begin. How well does this match the Fraunhofer boundary given in class?

20.z  a) Phased slit diffraction. Imagine a vertical slit of width \( a \), illuminated in a strange way: for \( 0<x'<a/2 \), the aperture field is \( E(x') = 1 \) and for \( -a/2<x'<0 \), \( E(x') = -1 \) (\( \pi \) out of phase, caused for example by a piece of glass of the right thickness that covers half the slit). Find a function that is the shape of \( E(\theta_z) \) (you can ignore leading constants and phase factors). Plot the shape of intensity \( I(\theta_z) \) vs. \( \theta_z \) in radians, for the case \( a = 500\lambda \).

b. Justify physically from phases why \( E(\theta_z = 0) = 0 \) in c), and show how this occurs from your function in c), using L’Hospital’s rule or limits for small \( \theta \).

H21. P10.5,6,7,11.5 Note: on all of these, since they are Fraunhofer diffraction, give the answers in angular form \( I(\theta) \), or \( I(\theta_x,\theta_y) \), and the plots as well (in radians), not as directed in the text. On 11.7, ignore the sentence that talks about a lens...not needed if you use the angular form.
H22. (Thurs) P11.3, 22.x, y

22x Fresnel Zones (8pts)

a) The semicircular phasor drawing below shows $E$ at the center of a screen as a circular aperture is opened more and more, until the first zone is open. Finish the sketch for the zones 2, 3, 4, 5, 6, using the "vibration curve" at the right as a guide for the spiraling that is due to increasing $R$ and the obliquity factor. Will 6 zones filling the aperture give you a bright or dark spot? What is the intensity at the screen center compared to the incident intensity (use the length you get from the

b) Explain why the distance from starting point to the center of the spiral is the strength of the incident $E$-field, $E_0$, that you get if you remove all apertures.

c) Using measurements from the vibration curve diagram, find the length of three phasors that represent light field $E$ at the screen center from zones 1,3,5 respectively, in terms of $E_0$. If you made a zone plate that blocked zones 2,4,6 as well as all zones $n>6$, what would the approximate $E$ field at the center of the screen be in terms of $E_0$? What would the approximate intensity be of the light at the screen center compared to the intensity of the laser beam with no aperture (this zone plate is now acting as a device to focus light).

d) Repeat the previous question with a plate made to block zones 1,3,5 and $n>6$, but allow transmission through zones 2,4,6.

e) A circular aperture has a diameter of $40\lambda$. How far away on the axis (in units of $\lambda$) does your screen need to be to have the Fresnel zones 1 to 6 fill the aperture? Note: in the Pythagorean triangle you set up, one side would be the radius, not the diameter. What is the width of the 6th zone ($x_6 - x_5$)?

22y. When the screen distance $z$ is large enough (or the aperture small enough), only one Fresnel zone fills a circular aperture of diameter $D$. Find that $z$. It should match the approximate $z$ we got in class for the boundary between Fresnel and Fraunhofer diffraction. For bigger $z$’s, there’s no way to make the center of the screen dark.

**Boundaries of Fresnel zones:**
23.x:
A He-Ne laser beam $\lambda=632$ nm has a beam waist of 1 mm at the center of a cavity.

a) At what distance (from the center) will the beam radius be 2 mm, and what is the radius of curvature of the wavefronts here?
b) What is the divergence angle of the laser?
c) Choose a lens ($f$) and its position from the center so that we will have a new beam waist of 0.1 mm at the focus. There are many possible answers, but show why your choices result in this beam waist. For this problem, assume that the lens diameter is always larger than the beam diameter, so that the lens diameter is not limiting the light beam that you focus.

23y. A He-Ne laser beam $\lambda=632$ nm laser cavity has mirrors with radii 1m and 2 meters

a) Find the beamwaist, Rayleigh range, and focus position (from the mirror with radius 1m) when the length $L$ of the cavity is 2.5m.

b) Plot the beamwaist vs $L$ over the entire stable region of $L$’s.

24.x Longitudinal laser modes.

a. Derive the simple relation between longitudinal mode number and allowed frequency $\nu_m$ given half-wavelengths fit between the two mirrors. For a HeNe laser, $\lambda=632$ nm, find the approximate m-value when $L=0.5$m.

b. If the gain bandwidth is $\Delta \nu_{\text{gain}}=1.5$ GHz FWHM (from Doppler broadening of a hot plasma) find how many longitudinal modes are lasing. Find the coherence length of the light that is emitted ($\Delta \nu_{\text{gain}}$ gives the frequency spread).

c. If we use a Fabry-Perot interferometer, we can select just one of the longitudinal modes to lase. The width of this single-$m$ line is determined by the leakage time of light from the cavity, $\Delta \nu_{\text{line}} = \frac{C}{2L} (1-R)$, where $R$ is the reflectance of the output coupler. Use $R = 99.0\%$. Find the coherence length of the light that is emitted.

d. Estimate the photons/second that pass a given point in the exterior laser beam if it has a diameter of 2mm and a power 1 mW. Now estimate the total number of photons inside the laser (roughly constant). You can get this knowing that 1% of the photons get through the output coupler each time they strike it, and also deducing how many times a second each photon inside strikes the output coupler.

24y. Vacuum modes and blackbody radiation

a. Compare two frequency regions near two different visible photon energies: $h\nu_a = 2$ eV and $h\nu_b = 3$ eV. Which region (2eV or 3 eV) has the most E/M wave modes per hertz per m³, and by what factor?

b. What are the average numbers of photons in each mode and also the average energy/mode at 2 eV and 3 eV if they are in equilibrium with the surface of the sun at $kT \approx 1$ eV?
c. Using the above information, which region (2eV or 3eV) has the greatest intensity of blackbody radiation from the sun (proportional to average energy/hertz), and by what factor?

24z. Calculate the approximate steady-state temperature of the earth:

a. Find the solar intensity at the earth’s surface. Use the power from P13.1, and put it into a spherical area of radius earth-sun distance.

b. Assume the earth absorbs all the energy from the sun that hits it, using the cross-sectional area of the earth, and find this power absorbed (the earth absorbs as though it were a disc facing the sun).

c. In steady state, the power absorbed from the sun equals the power emitted by the earth into space. Find the temperature at which the earth emits this power. For emission, it emits outward in all directions so use the surface area of a sphere. You should get something reasonable for the earth’s temperature. Note: Reality is a little more complicated…a full model would include reflection from the earth’s surface, as well as reflection back from greenhouse gases, the emissivity, and all of these as a function of frequency or wavelength.