Homework assignments are due at 11 pm (building close).

Labs are shown in red

H1. P0.1,3,6,14,18,20 Hint: on 18 use eq 0.30. If you need more examples with complex numbers, google "complex numbers tutorial". For more examples on vector calculus, google "curl and divergence".

H2. P1.2,4,5 and problem 2x below. On 1.2, you can choose your coordinates so E is along any coordinate you want, and choose a k that is perpendicular to it, because you're given that E, k are perpendicular. For example, you can choose your coordinate system so that \( \vec{E}_o = E_o \hat{x}, \quad \vec{k} = k \hat{y} \). And remember \( \vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z \)

Prob2x. If you were to create an E-field \( \vec{E} = (7x^2y^3 \hat{x} + 2z^4 \hat{y}) \cos \omega t \) from a changing charge distribution and currents in looped wires, find:

a. \( \rho(x,y,z,t) \)

b. \( \frac{\partial \vec{B}(x,y,z,t)}{\partial t} \)

c. Find the B and J that are consistent with this and show that E is a solution to the wave equation eqn 1.40. [Note: when you find B and J consistent with all of Maxwell's equations, then E will always be a solution to the full wave equation 1.40. So in showing it obeys the wave equation, you're showing that the wave equation satisfies Maxwell's equations in this particular case].

d. Some of the J will come from changing \( \rho \), and some you will have to supply from current in neutral wires. Find the terms in J(x,y,z) you would have to supply in wires (the divergence free terms).

H3. P1.6, 9,11, L1.10 (8 pts), and Prob 3x below.

Prob3x. a) An electromagnetic wave is traveling in the (2, -1, 3) direction, and its E-field oscillates in the (1, 2, 0) direction. Note that neither of those vectors has been normalized. The wave’s electric field amplitude is 5 V/m, its wavelength is 13 m, and its speed is obviously \( 3 \times 10^8 \) m/s. Write down an exponential plane wave which describes the electric field of the wave. b) Add a phase shift so that at \( t = 0 \) and \( \vec{r} = 0 \) the real part of E is \( \frac{1}{2} \) its maximum.

H4. P2.3 (8 pts) 5,7 On 2.3 the amplitude of the charge displacement will be much less than an angstrom. A good one to use Mathematica on. You know you’re doing it right if most of the values of \( n \) and \( \kappa \) at the end are between 0.1 and 2.

H5. P2.10, 3.1,2,5. On 3.1, just derive the far right form of the eqns). On 3.2, to make it easier, just do it for two rows of equations, not all 4.

H6. L3.4 (8 pts), P3.10, 12 On P3.12, also answer: b) what is the wavelength of the evanescent wave that propagates along the surface?

H7. P3.13, P4.1,2,3,4. On 2, add: b) (4 pts also) give an explanation with Phys 123 concepts and skills why the maxima and minima occur at one or two wavelengths you see. It's easiest to explain where R is large and small, which corresponds to where T is small and large.
H8. P4. 5,6,8,10

H9. L4.11 (8 pts), P4.15, 17. On L4.11, also find big F, and from F find R for each coating, assuming the same R on both layers. The easiest way to measure FWHM is to consider the distance on the screen between peaks as a phase shift of $2\pi$. Then you measure the peak FWHM as a fraction of $2\pi$, convert to radians, and use eqn 3.48.

On 17, you can just try all the possibilities, but I wrote a simple double loop to test all the combinations of materials to find the lowest R.

H10. P5.2, and (phase matching problem: 2013 edition, R40 on pg 222, same as 2011 edition, R28 on pg 143), and 10x below. On R40, the physics is that all the n's grow with frequency, so an index "ellipses" at one frequency might intersect an index "spheres" at another frequency. You can do this with polar plots in $\theta$, but maybe the easiest way is to plot vs $\theta$: normal plots n-e($\omega, \theta$), n-e(2$\omega, \theta$), n-o(\omega), n-o(2$\omega$). Of course the n-o's don't depend on $\theta$, and will be straight lines. Look for intersections of an n-e at one frequency with an n-o at another frequency.

10x. Light from air enters a uniaxial crystal with the optical axis along the z axis shown. The y direction is into the page. The optic axis is 45 degrees to the crystal surface. The incident k-vector is in red:

$$\vec{k}_{\text{inc}} = \frac{k}{\sqrt{5}} (2\hat{x} - \hat{z})$$.

The E-field (double arrows) is in the plane of the figure with direction $$\vec{E}_{\text{inc}} = \frac{E_o}{\sqrt{5}} (\hat{x} + 2\hat{z})$$. The optical properties of the crystal are given by $\chi_x = 3, \chi_y = 3, \chi_z = 8$.

a) Find the numerical values for $n_e, n_o$, and plot $n_e(\theta_{\text{OA}})$ as a function of the angle between the OA and the unknown $k_i$ in the crystal. Also plot $n_e(\theta_i)$ where $\theta_i$ is measured from the crystal normal.

b) Find $\theta_i$ in degrees from the crystal normal.

c) Find the angle $\theta_i$ from Snell’s law (measured from the crystal normal), i.e. find where $\sin \theta_i$ (a constant) and $n_e(\theta_i)\sin \theta_i$ are equal. What is $\theta_i$ from the OA? Check: I got 57deg from OA axis.

d) Knowing $\theta_i$, write a unit vector for $\vec{k}_i$ (in the x,z coordinate system).

e) From $\vec{k}_i \cdot (\varepsilon_r \vec{E}_i + \vec{P}) = 0$ find the ratios of the components of E in the crystal, and hence the direction of $\vec{E}_i$ in the crystal coordinate system. Find the angle between E and $\vec{k}_i$.

f) Knowing that S and E are perpendicular, find the angle that S makes with the optic axis. Make a sketch showing E, K and S in the crystal. Check: I got S is 34 degrees from the OA.

H11. P6.1,2,4

H12. P6.5,6,7, and 12.x below.
12.x. An initial light state is described by \( \begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix} \). x is the horizontal direction, y vertical.

a) Describe the initial state of the light in as much detail as possible.

b) It strikes a quarter-wave plate with its fast along the y axis, and then a linear polarizer that transmits at -45 degrees from the +x axis. What fraction of the initial intensity is transmitted? Use Jones matrices.

c) Just before the final polarizer, describe the state of the light in as much detail as possible.

H13. P7.1,3,4 and 13x (12 pts) below.
13x: In class you will borrow three linear polarizers to be returned on Friday. Complete the polarizer activity. You can do this alone, or in pairs if you do all the activities together.

H14. P0.21,23,24 (Fourier theory), P7.5

H15. P0.26,27, 28,15x.

15x. Continues 28 above. a) Find the Fourier transform of a \( f(t) \) consisting of three Gaussian optical pulses of the form 
\[ e^{-t^2/2T^2} \sin \omega_o t, \] 
but separated by a time \( t_1 \). The three Gaussians are centered at \( t=0, \ t=t_1, \) and \( t=-t_1 \).

Hint: Use a convolution theorem. The three-pulse function \( f(t) \) is a convolution of \( e^{-t^2/2T^2} \sin \omega_o t \) with three delta functions. You will use the results of 28 above. Here is an easy check: when \( t_1=0 \), the three pulses are on top of each other, so you should get 3x the answer in 28.

b) For \( \omega_o = 1, \ T = 8, \ t_1 = 30 \), plot \( f(t) \). Plot the imaginary part of \( f(\omega) \).

Comment on how your plot illustrates concepts we’ve learned about Fourier transforms of pulses.

H16. P7.7,8 only

H17. P8.2,4, 8.9a, and 17x,17y, and parts below

On 8.4 add: b) Find \( t_c \) from eq. 8.13. Discuss whether it obeys the uncertainty principle discussed in class. c) From eq. 8.16, find the fringe visibility at \( \tau = t_c \) and \( 2t_c \).

On 8.9a, skip b) and add: part c below

17x: Coherence of starlight

The nearest star (other than our sun) to us is Proxima Centauri at a distance of 30 trillion kilometers, and it has an angular diameter of 2 millionth of a degree or 7 milliarcseconds (1 milliarcsecond is 1 thousandth of an arcminute which is one sixtieth of a degree…thanks to the Babylonians, who loved multiples of 60). The only reason we know its diameter is because of interferometry.

a. If an astronomer filters the white starlight through a green filter, \( \lambda = 500 \text{ nm}, \ \Delta \lambda = 50 \text{ nm} \), what are the approximate temporal coherence length and coherence time of the light after passing through the filter? Use the uncertainty principle.

b. Using concepts from class, sketch the power spectrum \( I(\omega) \) for this light, and label the approximate width in rad/sec, as well as the position of the peak in rad/sec.

c. Using concepts from class, sketch the interferogram \( I(\tau) \) for delay \( \tau \) for this single star. Label the approximate width of the wiggles in femtoseconds (fs), as well as the approximate coherence time.
d. What is the approximate spatial coherence length of this light due to the diameter of the star? Use the simple estimate \( h_c = \frac{\pi}{\theta_{\text{edge}} k} \). Use a single \( \lambda = 500 \text{ nm} \) for this analysis. This length is approximately the baseline of a stellar interferometer needed to resolve the star’s diameter.

17y: In 8.9a) Instead of a line source with constant intensity, we consider a 2-D disc with constant intensity like the sun. We can still do a one-dimensional problem, if we use for \( \gamma' = \frac{a}{2} \), \( I(\gamma') = I_0 \sqrt{1 - \frac{4 \gamma^2}{a^2}} \), in other words, the intensity is proportional to the width of the disc as we move along \( y \), so the effective 1-D intensity is more concentrated near \( y=0 \) than for a line source. For \( \gamma' > \frac{a}{2} \), \( I = 0 \).

i) Find \( \gamma(h) \) in terms of special functions...do it it Mathematica. ii) change to the variable \( u = \frac{a k h}{2 R} \), plot \( \text{Re}(\gamma(h)) \), and also plot \( \text{Re}(\gamma(h)) \) for the line source in a). Which has a longer spatial coherence length \( h_c \), the line or disc source? Why does that make sense?

H18. P 9.1,(2013:19.4 or 2011:9.3) and (2013:9.3 ...see here if you have 2011) and 18x:

18x: The text’s version of the eikonal equation \( \nabla R = n(\hat{R}) \cdot \hat{s}(\hat{R}) \) is usually written in another form: we eliminate \( R \) by differentiating, and get \( \frac{d}{ds} (n \hat{s}) = \nabla n \). This says that the gradient of \( n \) determines how the direction \( \hat{s} \) of the ray changes as you move a distance \( ds \) along the path. This form can be used in a computer solution to trace any ray’s path.

Let’s restrict \( n \) so it changes in only one direction, for example \( y \). Then we can write \( \nabla n = n' \hat{y} \). If the direction of the ray in the x-y plane is given by \( \theta \), the direction is the unit vector \( \hat{s} = \cos \theta \hat{x} + \sin \theta \hat{y} \).

a) show that \( \frac{d}{ds} (n(y) \hat{s}) = \nabla n(y) \) becomes \( d \theta = \frac{1}{n} n' \cos \theta ds \). \( s \) is the variable that tells you where you are along the ray path.

Hint: \( \frac{d}{ds} = \frac{d \theta}{ds} \frac{d}{d \theta} \). After differentiating, dot both sides with some unit vector that will give you a scalar equation in terms of \( d \theta \).

b) Argue (including a sketch of how a couple of rays change direction ) that \( d \theta = \frac{1}{n} n' \cos \theta ds \) means if the gradient \( n' \) doesn’t change sign, then rays not aligned with the gradient will eventually end up being aligned with the gradient (y axis), if they travel long enough.

H19. (2013: P9.6,7,8,9 or 2011: P9.5,6,7,8)


21.x Thin lens addition formula. Using any theoretical method you like, join two (or any number of) thin lens together without any space between them, and show that \( 1/f_{\text{total}} = 1/f_1 + 1/f_2 \). Note: this is why opticians use "optical power" = \( 1/f \) for a lens, because they can add the "powers" very easily from the many test lenses to get the final prescription.
21.y Spherical wave addition. Our diffraction analysis methods all rely on Huygen's principle of adding E-fields from spherical waves at every point in an aperture. Here we will consider just three point sources at \( y' = a, 0 \) and \(-a\). A screen is placed at \( z = d \). Each source emits at the same phase with wavelength \( \lambda \).

\[ a) \] Scalar addition approximation: Write the sum \( E(y, d) \) if each source emits a scalar wave \( A e^{ikR} \). Plot the shape of \( I(y, d) \propto E \) if \( a = 100\lambda \), for \( d = a \) and \( d = 1.5a \).

b) Fresnel (near field) regime. You need \( d \gg a \) so that the angles to the screen are small. Keep doubling \( d \) and look at the patterns. As long as the shapes keep changing (not just expanding), this is the Fresnel regime. Plot two of these that look pretty different and give the \( d \)'s.

c) Fraunhofer (far field) regime. Keep doubling \( d \) until you find a case where the shape of the pattern doesn't change (just expands with \( d \), so it's the same angular form). Plot two of these that look similar for very different \( d \)'s and give an approximate \( d \) where you see the Fraunhofer regime begin. How well does this match the Fraunhofer boundary given in class?

H22. (2013: P10.6,7 or 2011: P10.8,9) and 11.5 and 22.x

22.x a) Phased slit diffraction. Imagine a vertical slit of width \( a \), illuminated in a strange way: for \( 0 < x' < a/2 \), the aperture field is \( E(x') = 1 \) and for \(-a/2 < x' < 0\), \( E(x') = -1 \) (\( \pi \) out of phase, caused for example by a piece of glass of the right thickness that covers half the slit). Find a function that is the shape of \( E(\theta_x) \) (you can ignore leading constants). Plot the shape of intensity \( I(\theta_x) \) vs. \( \theta_x \) in radians, for the case \( a = 500\lambda \).

b. Justify physically from phases why \( E(\theta_x = 0) = 0 \) in c), and show how this occurs from your function in c), using L'Hospital's rule or limits for small), using L'Hospital's rule or limits for small \( \theta \).

H23. (Thurs) P11.3,7, 23x below

23x Fresnel Zones

a) The semicircular phasor drawing below shows \( E \) at the center of a screen as a circular aperture is opened more and more, until the first zone is open. Finish the sketch for the zones 2, 3, 4, 5, 6, using the "vibration curve" at the right as a guide for the spiraling that is due to increasing \( R \) and the obliquity factor. Will 6 zones filling the aperture give you a bright or dark spot?

b) Explain why the distance from starting point to the center of the spiral is the strength of the incident E-field, \( E_o \), that you get if you remove all apertures.

c) Using measurements from the vibration curve diagram, find the length of three phasors that represent light field \( E \) at the screen center from zones 1,3,5 respectively, in terms of \( E_o \). If you made a zone plate that blocked zones 2,4, 6 as well as all zones \( n>6 \), what would the approximate E-field at the center of the screen be in terms of \( E_o \)? What would be the approximate intensity be of the light compared to the intensity of the laser beam with no aperture (this zone plate is now acting as a device to focus light).

d) Repeat the previous question with a plate made to block zones 1,3,5 and \( n>6 \), but allow transmission through zones 2,4,6.
A circular aperture has a diameter of 40\(\lambda\). How far away on the axis (in units of \(\lambda\)) does your screen need to be to have the Fresnel zones 1 to 6 fill the aperture? Note: in the Pythagorean triangle you set up, one side would be the radius, not the diameter. What is the width of the 6th zone (\(x_6 - x_5\))?

**H24. P11.1, 24x, 24y**

24.x: A He-Ne laser beam \(\lambda = 632\) nm has a beam waist of 1 mm at the center of a cavity.

a. At what distance (from the center) will the beam radius be 2 mm?

b. What is the radius of curvature of the wavefronts that intersect the z axis here?

c. What is the half-angle of divergence far from the focus, in rad, degrees?

d. If you put a lens of 10 cm focal length at the point where the beam radius is 4 mm, what is the new divergence angle, determined by the focusing? Ignore the previous weak divergence.

e. Find the new Rayleigh range and the new beam waist for the above question.

24y. A He-Ne laser beam \(\lambda = 632\) nm laser cavity has mirrors with radii 1m and 2meters

a. Find the beamwaist, Rayleigh range, and focus position (from the mirror with radius 1m) when the length L of the cavity is 2.5m.

b. Plot the beamwaist vs L over the entire stable region of Ls.
25x Longitudinal laser modes.

a. Derive the simple relation between longitudinal mode number \( m \) and allowed frequency \( \nu_m \) given half-wavelengths fit between the mirrors. For a HeNe laser, \( \lambda = 632 \) nm, find the approximate \( m \)-value when \( L = 0.5 \) m.

b. If the gain bandwidth is \( \Delta \nu = 1.5 \) GHz FWHM (from Doppler broadening) find how many longitudinal modes are lasing. Find the coherence length of the light that is emitted.

c. If a grating is used to feed back light into the laser we can force only one \( m \) to lase. If the grating line spacing is 200 lines/mm, find the difference in diffraction angle between two successive \( m \)'s. You can use the small angle result without too much error: \( h \sin \theta = n \lambda \) becomes \( h \theta = \lambda \), so \( \Delta \theta = \Delta \lambda / h \). You will find the difference in angle to be very small (would not be resolvable by typical spectrometer), but the stimulated emission and feedback is so nonlinear that one angle will be preferred, and hence one mode.

d. When the HeNe is grating-stabilized, the width of this single-\( m \) line is determined by the leakage time of light from the cavity, \( \Delta \nu_{\text{line}} = \frac{c}{2L} (1 - R) \), where \( R \) is the reflectance of the output coupler. Use \( R = 99.0\% \). Find the coherence length of the light that is emitted.

25y Vacuum modes and blackbody radiation

a. Compare two frequency regions near two different visible photon energies: \( h \nu_a = 2 \) eV and \( h \nu_b = 3 \) eV. Which region (2eV or 3eV) has the most E/M wave modes per hertz per m\(^3\), and by what factor?

b. What are the average numbers of photons in each mode and also the average energy/mode at 2 eV and 3 eV if they are in equilibrium with the surface of the sun at \( kT \approx 1 \) eV?

c. Using the above information, which region (2eV or 3eV) has the greatest intensity of blackbody radiation from the sun (proportional to average energy/hertz), and by what factor?

25z Calculate the approximate steady-state temperature of the earth:

a. Find the solar intensity at the earth’s surface. Use the power from P13.1, and put it into a spherical area of radius earth-sun distance.
b. Assume the earth absorbs all the energy from the sun that hits it, using the *cross-sectional* area of the earth, and find this power absorbed (the earth absorbs as though it were a *disc* facing the sun).

c. In steady state, the power absorbed from the sun equals the power emitted by the earth into space. Find the temperature at which the earth emits this power. For emission, it emits outward in all directions so use the surface area of a sphere. You should get something reasonable for the earth’s temperature. Note: Reality is a little more complicated…a full model would include reflection from the earth’s surface, as well as reflection back from greenhouse gases, the emissivity, and all of these as a function of frequency or wavelength.