10x. Light from air enters a uniaxial crystal with the optical axis along the \( z \) axis shown. The \( y \) direction is into the page. The optic axis is 45 degrees to the crystal surface. The incident \( k \)-vector is in red: \( \vec{k}_{inc} = \frac{k}{\sqrt{5}}(2\hat{x} - \hat{z}) \). The \( E \)-field (double arrows) is in the plane of the figure with direction \( \vec{E}_{inc} = \frac{E_n}{\sqrt{5}}(\hat{x} + 2\hat{z}) \). The optical properties of the crystal are given by \( \chi_x = 3, \chi_y = 3, \chi_z = 8 \).

a) Find the numerical values for \( n_o, n_e \), and plot \( n_e(\theta_{k-OA}) \) as a function of the angle between the OA and the unknown \( k \) in the crystal. Also plot \( n_e(\theta) \) where \( \theta \) is measured from the crystal normal.

b) Find \( \theta_i \) in degrees from the crystal normal.

c) Find the angle \( \theta_f \) from Snell’s law (measured from the crystal normal). i.e. find where \( \sin \theta_f \) (a constant) and \( n_e(\theta_f)\sin \theta_f \) are equal. What is \( \theta_f \) from the OA? Check: I got 57deg from OA axis.

d) Knowing \( \theta_i \), write a unit vector for \( \vec{k}_i \) in the crystal’s coordinate system.

e) From \( \vec{k}_i \cdot (\varepsilon_e \vec{E}_i + \vec{P}) = 0 \) find the ratios of the components of \( E \) in the crystal, and hence the direction of \( \vec{E}_i \) in the crystal coordinate system. Find the angle between \( E \) and \( \vec{k}_i \).

f) Knowing that \( S \) and \( E \) are perpendicular, find the angle that \( S \) makes with the optic axis. Make a sketch showing \( E \), \( K \) and \( S \) in the crystal. Check: I got \( S \) is 34 degrees from the OA.