1. With the sun at the horizon, light goes through more atmosphere to reach us, scattering more of the higher frequency colors out of it, leaving it more red.

2. A pigment that transmits blue (absorbs other colors), and something that scatters white light (size greater than \( \lambda \)), usually.

3. In an arc at 90 degrees from the line between you and the sun. I accepted any position along this arc: i.e. N or S at the horizon, or above you.

b) Your answer had to be consistent with the position you chose in a). You have to use the idea that a dipole doesn’t radiate along the line of its acceleration. If you chose N or S, the V light scatters toward you...H light does scatter, but it goes mostly up and down from the molecule you are looking, not toward you. If you chose above you, then the molecules there scatter mostly H light toward you, because they move parallel to the horizon. They scatter V light in the N and S directions, but those don’t go to you.

4. 

\[
\begin{bmatrix}
\cos^2 \theta & \sin \theta \cos \theta \\
\sin \theta \cos \theta & \sin^2 \theta 
\end{bmatrix}
\begin{bmatrix}
A \\
B e^{i\phi}
\end{bmatrix}
- \left( \begin{bmatrix}
\cos \theta (A \cos \theta + B e^{i\phi} \sin \theta) \\
\sin \theta (A \cos \theta + B e^{i\phi} \sin \theta)
\end{bmatrix}
\right)
\]

\[
I - \frac{1}{2} n ce_0 |E_{eff}|^2 (\cos^2 \theta + \sin^2 \theta)^2 |A \cos \theta + B e^{i\phi} \sin \theta|^2
\]

\[
= \frac{1}{2} n ce_0 |E_{eff}|^2 \left[ A^2 \cos^2 \theta + AB \left( e^{i\phi} + e^{-i\phi} \right) \sin \theta \cos \theta + B^2 \sin^2 \theta \right]
\]

\[
= \frac{1}{2} n ce_0 |E_{eff}|^2 \left[ A^2 \cos^2 \theta + AB \cos \phi \sin 2\theta + B^2 \sin^2 \theta \right]
\]

\[
\frac{\partial I}{\partial \theta} = \frac{1}{2} n ce_0 |E_{eff}|^2 \left[ -2A^2 \cos \theta \sin \theta + 2AB \cos \phi \cos 2\theta + 2B^2 \sin \theta \cos \theta \right] = 0
\]

\[
\Rightarrow -2A^2 \cos \phi \sin \theta + 2AB \cos \phi \cos 2\theta + 2B^2 \sin \theta \cos \theta = 0
\]

\[
\Rightarrow 2AB \cos \phi \cos 2\theta - (A^2 - B^2) \sin 2\theta = 0 \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2AB \cos \phi}{A^2 - B^2}
\]

\[
\Rightarrow \tan 2\theta = \frac{2AB \cos \phi}{A^2 - B^2} \Rightarrow 2\theta = \tan^{-1} \left( \frac{2AB \cos \phi}{A^2 - B^2} \right)
\]

5. a) 

\[
\begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
= \begin{bmatrix}
\cos 2\theta \\
\sin 2\theta
\end{bmatrix}
= \begin{bmatrix}
\cos 120 \theta \\
\sin 120 \theta
\end{bmatrix}
\]

which is light linearly polarized at 120 deg from the x axis.

You can also use the idea that the purpose of a half wave plate is to rotate the polarization an angle of 2\( \theta \) about the fast axis.

b) 

\[
\begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{bmatrix}
\begin{bmatrix}
1 \\
i
\end{bmatrix}
= \begin{bmatrix}
\cos 2\theta + i \sin 2\theta \\
\sin 2\theta - i \cos 2\theta
\end{bmatrix}
\]

You can do this with numbers or symbols, but the goal is to get the 

relative phase between the top and bottom elements. 

\[
\begin{bmatrix}
\cos 2\theta + i \sin 2\theta \\
\sin 2\theta - i \cos 2\theta
\end{bmatrix}
= \begin{bmatrix} e^{i2\theta} \\
-ie^{i2\theta}
\end{bmatrix}
= e^{i2\theta} \begin{bmatrix} 1 \\
-i
\end{bmatrix}
\]

So it’s now R-circ polarized.

6. a) 

\[
\nu_p = \frac{\omega}{k} = \frac{c}{n} = \frac{c}{n_n + b\lambda^2}
\]
b) Several ways to go from here. You can do the derivative in $\lambda$ or $\omega$.

\[
v_g = \frac{\partial \omega}{\partial k} = \left[ \frac{\partial k}{\partial \omega} \right]^{-1} = \left[ \frac{\partial}{\partial \omega} \left( \frac{n_o + b(2\pi c)^3 / \omega^3}{c} \right) \right]^{-1}
\]

\[
= \left[ \frac{n_o + b(2\pi c)^3 / \omega^3 - 3\omega b(2\pi c)^3 \omega^{-4}}{c} \right]^{-1} = \frac{c}{n_o - 2b(2\pi c)^3 / \omega^3}
\]

Here it is differentiating with $\lambda$.

\[
v_g = \left[ \frac{\partial \lambda}{\partial \omega} \right]^{-1} = -\left[ \frac{\partial \lambda}{\partial \omega} \right]^{-1}
\]

\[
\frac{\partial \lambda}{\partial \omega} = \frac{\partial}{\partial \omega} \left( \frac{2\pi c \lambda}{\omega^2} \right) = \frac{2\pi c}{\omega^2} - \frac{2\pi c}{\omega^2} = -\frac{\lambda^2}{2\pi c}
\]

\[
v_g = \left[ \frac{\partial k}{\partial \lambda} \right]^{-1} = \left[ 2\pi \left( \frac{n_o + b\lambda^3}{\lambda^2} + \frac{3b\lambda^2}{\lambda} \right) \left( -\frac{\lambda^2}{2\pi c} \right) \right]^{-1} = \frac{c}{n_o - 2b\lambda^3}
\]

Has the right units!

7. a) $E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} dt = \frac{E_o}{\sqrt{2\pi}} \frac{e^{i\omega t}}{(i\omega)^2} (i\omega t - 1) \bigg|_{0}^{\tau} = \frac{E_o}{\omega^2 \tau \sqrt{2\pi}} \left( 1 + e^{i\omega t} (i\omega t - 1) \right).

b) E(t) has no oscillation, so E($\omega$) is centered close to $\omega=0$. I gave credit for lots of shapes, just so long as it was close to $\omega=0$. E($\omega$) will have some wiggles of spacing $1/\tau$ in the real and imaginary parts, from the factor $e^{i\omega t}$, which I haven't drawn. It has an envelope that decreases as $1/\omega^2$. But the most important things in the sketch is that it is centered about $\omega = 0$, and has a width of about $1/(\text{width in time space})$.

c) $f(t) = A \cos \omega_o t = A \left( \frac{e^{i\omega_o t} + e^{-i\omega_o t}}{2} \right)$, so.

\[
f(\omega) = \frac{A}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( \frac{e^{i\omega_o t} + e^{-i\omega_o t}}{2} \right) e^{i\omega t} dt = \frac{A}{2\sqrt{2\pi}} \left( \delta(\omega - \omega_o) + \delta(\omega + \omega_o) \right)
\]

d). $E(t) = E_o \frac{t}{\tau} \cos \omega_o t$, etc. This is a product in t-space that becomes a convolution in $\omega$ space: The integral over the delta functions gives a copy of part a) at both $\omega_o$ and $-\omega_o$.

\[
F \{ g(t)h(t) \} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\omega')h(\omega' - \omega) d\omega' = 1 \int_{-\infty}^{\infty} E_o \frac{t}{\tau} \left( \frac{1}{\sqrt{2\pi}} \frac{e^{i\omega t} (i\omega t - 1)}{\omega^2 \tau \sqrt{2\pi}} \right) \int_{-\infty}^{\infty} \delta(\omega' - \omega_o - \omega) + \delta(\omega' + \omega_o - \omega) d\omega' = \frac{E_o}{2(2\pi)^{3/2}} \frac{1}{\omega_o^2} \left( 1 + e^{i(\omega - \omega_o) t} (i(\omega - \omega_o) t - 1) \right) + \frac{1}{(\omega + \omega_o) t} \left( 1 + e^{i(\omega + \omega_o) t} (i(\omega + \omega_o) t - 1) \right)
\]
8. a) Dispersion, but specifically the second derivative of \( n(\omega) \). So it’s not sufficient that \( n \) depend on \( \omega \)…the group velocity must be different for different \( \omega \).

b) \( \tau_c \approx \frac{1}{\Delta\omega} \). When the pulses change due to broadening and chirping, that comes from phases shifts of the different frequencies relative to each other. But that doesn’t change the power spectrum, so the coherence time remains unchanged. The pulses in time are very different, but they interfere with each other the same.

9. Here I’m looking for the idea

\[
\frac{1}{\Delta\omega} = \frac{1}{\omega_{400\text{nm}} - \omega_{700\text{nm}}} = \frac{1}{\frac{2\pi c}{\lambda_1} - \frac{2\pi c}{\lambda_2}} = \frac{\lambda_1\lambda_2}{2\pi c(\lambda_2 - \lambda_1)}
\]

or \( \Delta\omega \approx \Delta\left(\frac{2\pi c}{\lambda}\right) = \frac{2\pi c}{\lambda_{\text{avg}}} \Delta\lambda \), gives a similar result

\[
l_c = c\tau_c \approx \frac{2\pi\lambda_{\text{avg}}^2}{\Delta\lambda} \approx 5000 \text{ to } 6000 \text{nm}
\]

So you would have to move one arm about half that, or about 3000 nm, i.e. a few optical wavelengths before the fringes start getting washed out.

Some of you need to work on the calculus of how differences or differentials work to relate \( \lambda, \omega, k \), as in this problem:

\[
\Delta\omega \approx \Delta\left(\frac{2\pi c}{\lambda}\right) = \frac{2\pi c}{\lambda^2} \Delta\lambda, \text{ never } \Delta\omega \approx \left(\frac{2\pi c}{\Delta\lambda}\right).\]

This ideas is also used in 6b). The only time that inverses of differentials appear are in uncertainty relations like \( \Delta t \approx \frac{1}{\Delta\omega} \), which don’t come from differentiating, but just from wave (Fourier) ideas.

10. a) Spatial coherence.

b) The visibility function of \( h \) comes from the FT of the source over all its angles. If the visibility goes to zero and then comes back as you increase \( h \), then the FT is showing “ringing”, similar to the sinc\(^2\) function. This ringing in the FT comes from a function (or its derivative) that goes to zero suddenly. So the source intensity doesn’t go to zero smoothly with angle (as in a Gaussian), but more suddenly as in a rectangular function (like a bar or disc of light), a triangular function, etc.