

Hatch 2-2427, Hess 2-2108

3 hour time limit. No books or notes.

$$\langle v \rangle = \frac{\Delta x}{\Delta t}$$

$$\langle a \rangle = \frac{\Delta v}{\Delta t}$$

$$v = v_o + a t$$

$$x = x_o + \frac{1}{2} (v_o + v) t$$

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$v^2 = v_o^2 + 2a (x - x_o)$$

$$\text{If } ax^2 + bx + c = 0, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$g = 9.80 \text{ m/s}^2$$

$$\Sigma F = ma \quad w = mg \quad f_s \leq \mu_s N \quad f_k = \mu_k N$$

$$W_{\text{net}} = KE_f - KE_i$$

$$KE_i + PE_i = KE_f + PE_f$$

$$KE_i + PE_i + W_{\text{nc,in}} = KE_f + PE_f + W_{\text{nc,out}}$$

$$W = F \cos \theta d \quad KE = \frac{1}{2} mv^2 \quad PE = mgh$$

$$PE = \frac{1}{2} kx^2 \quad F = -kx \quad P = W/t = Fv \cos \theta$$

$$p = mv$$

$$F \Delta t = \Delta p = mv_f - mv_i$$

$$x_{\text{cm}} = (\Sigma x_i m_i) / M$$

$$v_{\text{cm}} = (\Sigma v_i m_i) / M$$

$$\mathbf{F}_{\text{ext}} = M \mathbf{a}_{\text{cm}}$$

$$\mathbf{p}_i = \mathbf{p}_f$$

$$\langle \omega \rangle = \Delta \theta / \Delta t$$

$$\langle \alpha \rangle = \Delta \omega / \Delta t$$

$$\Delta s = r \Delta \theta$$

$$v = r \omega$$

Please write your **CID** \_\_\_\_\_

$$a_t = \Delta |v| / \Delta t = r \alpha$$

$$a_c = v^2 / r = r \omega^2$$

$$\Sigma F_c = m a_c$$

$$\omega = \omega_o + \alpha t$$

$$\Delta \theta = \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2 \alpha \Delta \theta$$

$$\Delta \theta = \frac{1}{2} (\omega_o + \omega) t$$

$$F = GMm / r^2$$

$$PE = -GMm / r$$

$$T^2 = (4\pi^2 / GM) r^3$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

$$R_{\text{earth}} = 6.38 \times 10^6 \text{ m}$$

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg}$$

$$\tau = F_{\perp} r = F r_{\perp}$$

$$\Sigma F = 0$$

$$\Sigma \tau = 0$$

$$I = \Sigma m_i r_i^2$$

$$I_{\text{sphere}} = (2/5) MR^2$$

$$I_{\text{hoop}} = MR^2$$

$$I_{\text{disk}} = \frac{1}{2} MR^2$$

$$I_{\text{rod (center)}} = (1/12) ML^2$$

$$I_{\text{rod (end)}} = (1/3) ML^2$$

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

$$\Sigma \tau = I \alpha$$

$$L = p_{\perp} r = I \omega$$

[ ? ] with choices means simple multiple choice. [ S ] with no choices means second significant digit. Only if an answer is truly zero (not just small) do you mark 0. Keep three significant digits throughout your calculations; do not round up to less than three. As an example, for 3.89, -0.00389, or  $3.89 \times 10^7$  you would mark 8, the second significant digit, on the **bubblesheet**.

Write your CID at the top of this exam, as well as on the Written Portion sheet. Did you do this? \_\_\_\_\_.

You lose 2 % if you don't.

A blue bumper car of mass 80 kg traveling right at 3 m/s hits a red bumper car of mass 120 kg moving left at 4 m/s. After the collision, the blue bumper car travels backwards at 2 m/s, and the red bumper car must be going [1S]\_\_\_\_\_ m/s to the [2?]\_\_\_\_\_ 1) right 2) left. In this collision, the kinetic energy change during the collision was [3S]\_\_\_\_\_ J.

$$p_i = p_f$$

$$80 \cdot 3 - 120 \cdot 4 = -80 \cdot 2 + 120 \cdot v_f$$

$$v_f = -0.667 \text{ m/s (left) } \mathbf{1 (6) 2(2)}$$

$$KE_i = \frac{1}{2} \cdot 120 \cdot 4^2 + \frac{1}{2} \cdot 80 \cdot 3^2 = 1320 \text{ J}$$

$$KE_f = \frac{1}{2} \cdot 120 \cdot (0.667)^2 + \frac{1}{2} \cdot 80 \cdot 2^2 = 187 \text{ J}$$

$$\Delta KE = 187 - 1320 = -1133 \text{ J } \mathbf{3(1)}$$

A boy on a skateboard with total mass 60 kg is traveling 10 m/s. A friend of mass 40 kg, who has no initial velocity, steps on the board as he passes by, and they travel together. Their speed right after the friend steps on is [4S]\_\_\_\_\_ m/s.

$$p_i = p_f$$

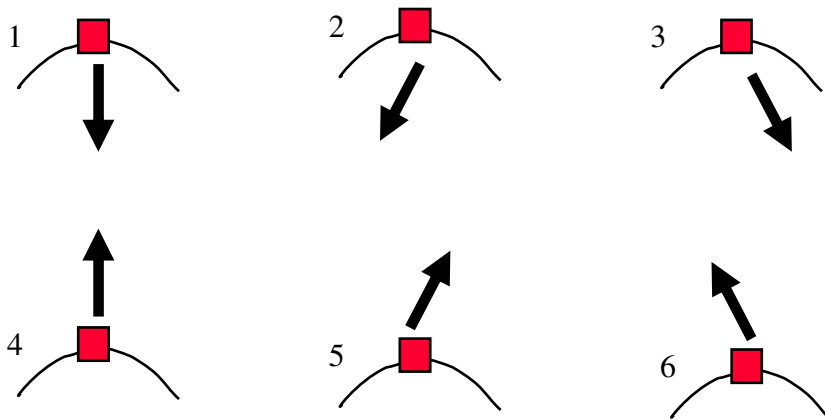
$$60 \cdot 10 = (60 + 40) \cdot v_f$$

$$v_f = 6.0 \text{ m/s } \mathbf{4 (0)}$$

A red car moves clockwise around a circle at constant speed at the instant shown. The total acceleration vector (or net force) is [5?]\_\_\_\_\_ (choose one picture below. If  $a=0$ , mark 0). If the car is moving clockwise and speeding up, the total acceleration vector is [6?]\_\_\_\_\_ (choose one picture below. If  $a=0$ , mark 0).

**acceleration toward center to turn 5 (1)**

**acceleration toward center plus tangential acceleration to speed it up. 6 (3)**



A grinding wheel has a moment of inertia of  $0.12 \text{ kg m}^2$ . It has an angular acceleration of  $5 \text{ rad/s}^2$ . If it accelerates constantly from rest, in 15 seconds it will have turned [7S]\_\_\_\_\_ revolutions. (Careful with units). The grinding wheel is a solid disk of radius 8 cm. It must have a mass of [8S]\_\_\_\_\_ kg. The torque that the motor puts on it to accelerate it is [9S]\_\_\_\_\_ Nm. When it is going fast enough, you turn off the motor. You now grind a piece of steel, and because of this it decelerates at  $0.3 \text{ rad/s}^2$ . The frictional force between the steel and the wheel is [10S]\_\_\_\_\_ N.

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \cdot 5 \cdot 15^2 = 562 \text{ radians} = 89.5 \text{ rev } \mathbf{7(9)}$$

$$I_{\text{disk}} = \frac{1}{2} MR^2$$

$$M = 2I/R^2 = 2 * 0.12 / (0.08)^2 = 37.50 \text{ kg. } \mathbf{8 (7)}$$

$$\Sigma \tau = I \alpha = 0.12 * 5 = 0.60 \text{ Nm } \mathbf{9(0)}$$

$$\Sigma \tau = I \alpha \quad \tau = Fr \quad F = \tau/R = I \alpha / R = 0.12 * 0.3 / 0.08 = 0.45 \text{ N } \mathbf{10 (5)}$$

A spaceship of mass 3000 kg is launched from the earth to a circular orbit in such a way that the speed of the space ship in orbit is 4000 m/sec. The radius of the orbit measured *from the center of the earth* is [11S]\_\_\_\_\_ m. While in orbit, the astronauts inside the ship [12?]\_\_\_\_\_ 1) are weightless 2) are in free-fall

$$F = GMm/r^2 = ma_c = mv^2/r. \quad r = \frac{GM_E}{v^2} = 6.67e-11 * 5.98e24 / (4e3)^2 = 2.49 \times 10^7 \text{ m } \mathbf{11 (4)}$$

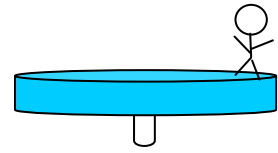
**12 (2)**

A merry-go round is spinning with a boy at the edge.

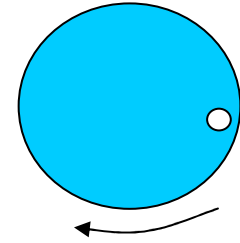
When the boy walks to the center the kinetic energy of the system

[13?]\_\_\_\_\_ 1) increases 2) decreases 3) stays the same. The rotational speed

[14?]\_\_\_\_\_ 1) increases 2) decreases 3) stays the same



A merry-go round is spinning with a boy at the edge. He slips off when he steps on a frictionless icy part of the merry-go-round. The rotational speed of the merry go round [15?]\_\_\_\_\_ 1) increases 2) decreases 3) stays the same



**13(1)** KE increases, just as in the skater example, because the boy has to put work into the system to get to the center

**14 (1)** Increases to conserve angular momentum as  $I_{\text{total}}$  drops

**15 (3)** The boy carries off his own angular momentum in  $L = p_{\perp} r$ , so the angular momentum of the disk doesn't change. In another view, he can't put a force on it while he slips, so he can't accelerate or decelerate it.

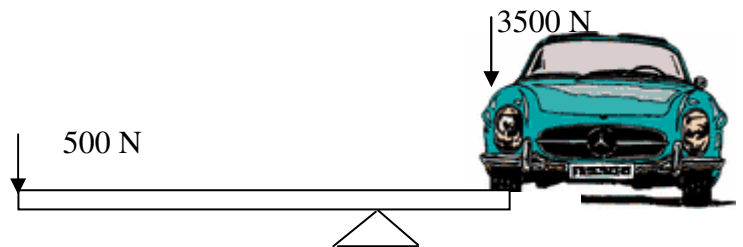
$$\Sigma \tau = 0$$

A man is trying to lift his car with a steel lever. The man's weight is 500 N, and the car's weight is 7000 N. He lifts only *two wheels* slightly off the ground. The other two are still on the ground. The fulcrum (pivot point) is 1 meter from the car's tires.

If he uses all his weight and stands on the end of the lever, the *total* length of the steel lever must be at least [16S]\_\_\_\_\_ m to lift the car. Neglect the weight of the lever.

$3500 \text{ N} * 1 \text{ m} = 500 \text{ N} * R$ . So  $R = 7.0 \text{ m}$ . So the total length must be at least 8.0 m (add the 1m).

**16 (0)**



A round object of radius 0.4 m, mass 8 kg and moment of inertia  $0.6 \text{ kg m}^2$  is rolling on the ground without slipping, and the speed of its center of mass is 9 m/s. The *total* kinetic energy of the object is [17S]\_\_\_\_\_ J. The round object rolls up an inclined plane without slipping and stops. A block, with the same mass and traveling at the same speed also slides without friction up the inclined plane and stops. The object that ends up highest on the inclined plane is [18?]\_\_\_\_\_ 1) the rolling object 2) the block 3) neither...same final height .

$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ . Because it rolls without slipping,  $v = \omega r$   $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = 1/2*8*9^2 + 1/2*0.6*(9/0.4)^2=476$  J. **17 (7)**

When the object rolls without slipping up the plane, all the KE is converted into PE. The block has only  $KE = \frac{1}{2}mv^2$ , so it can't go as far up. **18 (1)**

A firecracker at rest explodes into two pieces. A large piece moves to the right. A small piece moves to the left. Which piece moves faster [19?] \_\_\_\_\_ 1) the large piece 2) the small piece 3) the same speed. After the explosion, the center of mass of the two pieces combined [20?] \_\_\_\_\_ 1) is moving right 2) is moving left 3) is not moving.

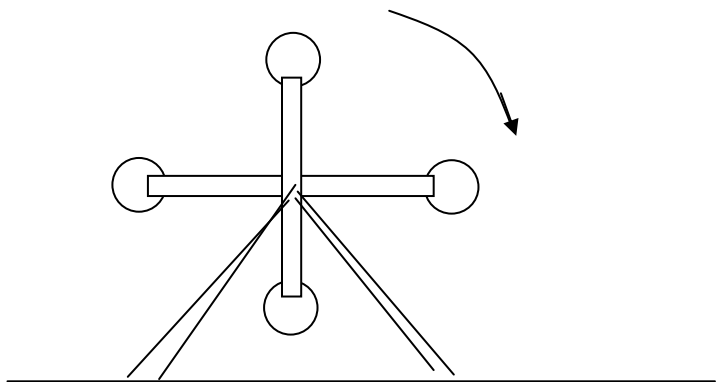
**19 (2)**, so that momentum is conserved (the two momenta are equal and opposite)  
**20 (3)** no external forces, so the center of mass can't accelerate.

The further a satellite is from the earth, the rotation period gets [21?] \_\_\_\_\_ 1) shorter 2) longer 3) doesn't change

**21 (2)**

A carnival ride has four cages on rods, rotating in a vertical plane. All the cages move at the same speed. If the tension in the rods is too great, a cage will break off. If the ride rotates too fast, the cage most likely to break off will be the one on the [22?] \_\_\_\_\_ 1) left 2) right 3) top 4) bottom 5) none; all equally likely

**22 (4)** At the bottom, the tension must equal  $mg$  plus the turning force. The turning force is the same for all the cages. So bottom breaks first.



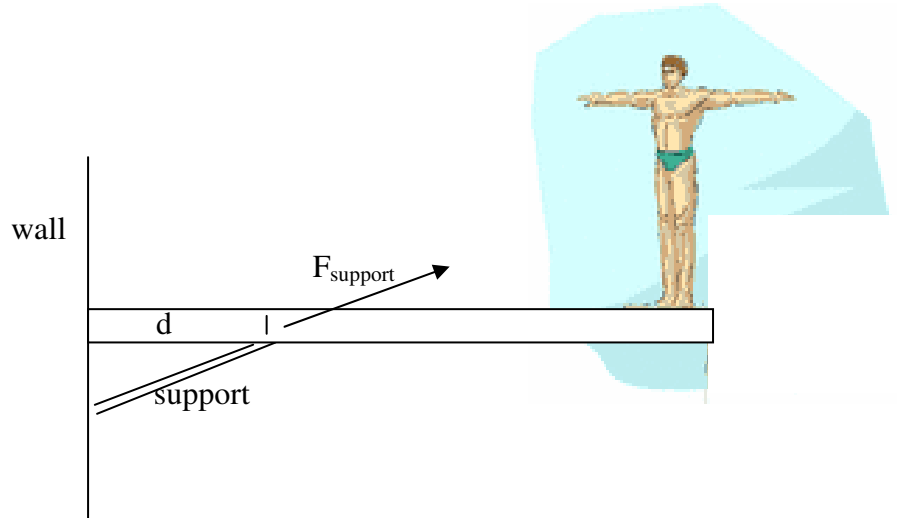
A diver stands on the end of a board weighing 800 N that is 4 meters long. The support below the board puts a force of 6000 N on the board a distance  $d=1.2$  m from the wall, along the line of the support, shown by the arrow, 30 degrees above the horizontal. The torque exerted by the support on the board about its left end is [23S]\_\_\_\_\_ Nm. The torque exerted by the weight of the beam on the board about its left end is [24S]\_\_\_\_\_ Nm. The weight of the diver must be [25S]\_\_\_\_\_ N.

$$\Sigma \tau = 0$$

$$\tau_{\text{support}} = F_{\perp} r = 6000 \text{ N} \sin(30) * 1.2 = 3600 \text{ Nm. } \mathbf{23 (6)}$$

$$\tau_{\text{board}} = F_{\perp} r = 800 \text{ N} * 2 \text{ m} = 1600 \text{ Nm. } \mathbf{24 (6)}$$

$$\Sigma \tau = 0, \text{ so } \tau_{\text{diver}} = 3600 - 1600 \text{ Nm.} = 2000 \text{ Nm} = W r = W(4\text{m}) \text{ so } W = 2000 \text{ Nm} / 4\text{m} = 500 \text{ N } \mathbf{25 (0)}$$

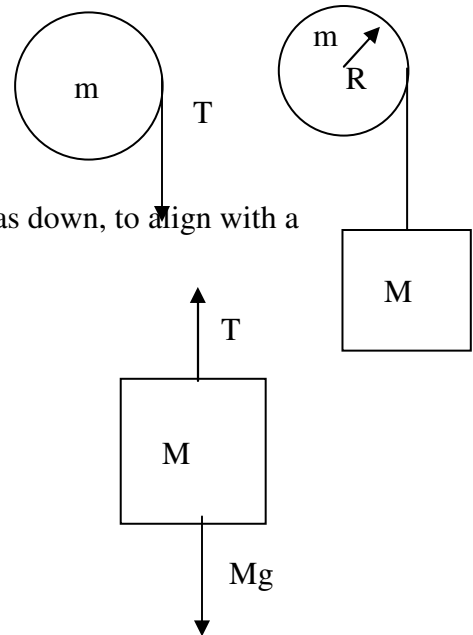


The disk of mass  $m$  has a rope wrapped several times around its edge. The falling weight has mass  $M$ . As the system accelerates, the disc rotates about its center. There is no slipping of the rope. For  $m=30$  kg, and  $M=50$  kg,  $R=20$  cm, the acceleration of the mass  $M$  is [27S]\_\_\_\_\_  $\text{m/s}^2$ .

**You can also put  $N$  and  $mg$  on diagram, but not required for motion.**

$$\Sigma \tau = I \alpha \rightarrow TR = \frac{1}{2} m R^2 \alpha$$

$$\Sigma F = Ma \rightarrow Mg - T = Ma \text{ Here I have put positive direction as down, to align with a}$$



Connection between  $a$ 's and  $\alpha$ :  $a = R\alpha$

**Putting these all together:**

$$Mg - T = Ma$$

$$Mg - \frac{1}{2} m R \alpha = Ma$$

$$Mg - \frac{1}{2} m a = Ma$$

$$a = \frac{Mg}{M + \frac{1}{2} m} = 9.80(50\text{kg} / (50 + 30/2)) = 7.538 \text{ m/s}^2 \quad \mathbf{(26-5)}$$