

# Testing non-classical theories of electromagnetism with ion interferometry

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We discuss using a table-top ion interferometer to search for deviations from Coulomb's inverse-square law. Such deviations would result from non-classical effects such as a non-zero photon rest mass. We discuss the theory behind the proposed measurement, explain which fundamental, experimentally controllable parameters are the relevant figures of merit, and calculate the expected performance of such a device in terms of these parameters. Such a device would be sensitive to deviations of a few times  $10^{-22}$  in the exponent of the inverse-square law, an improvement by five orders of magnitude over current experiments. It could measure a non-zero photon rest mass smaller than  $9 \times 10^{-50}$  grams, nearly 100 times smaller than current laboratory experiments.

The experimental search for deviations from current theories will eventually lead to the next, more fundamental theory of physics. Such studies challenge the Standard Model and give insight into the form of the underlying, more elemental theory. Coulomb's inverse-square law is the foundational law in electrostatics. Gauss's Law and Maxwell's equations are built upon this law and the principle of superposition. Precision tests of this law are essential to push forward our understanding of electromagnetism and its relation to the other forces.

Detection of any deviation from the inverse-square law would have far-reaching implications. Maxwell's equations would have to be modified along with much of the Standard Model. The notion that absolute electrostatic potential is arbitrary would have to be abandoned, along with many other ideas stemming from classical electromagnetism. Inverse square law violation would suggest a finite range for the electromagnetic force, implying a non-zero photon rest mass [1–3]. Consequences of a finite photon rest mass include frequency dependence of the velocity of electromagnetic waves in free space and a longitudinal polarization of photons [4]. Several grand-unification theories include massive photons [5, 6] and further measurement of the inverse square law can help confirm or disprove these theories.

Several studies have searched for consequences of a massive photon rather than testing the inverse-square law directly [2]. These studies involve many assumptions about the nature of interstellar space and the sources of the measured light waves. It is therefore necessary to verify these results with laboratory experiments where variables can be better controlled [19]. Furthermore, while the possibility of a massive photon supplies additional motivation to test Coulomb's law (and provides a common parameter by which experiments can be compared) it is possible that Coulomb's law is violated for reasons unrelated to photon rest mass. Only an experiment which specifically measures the inverse-square law would be sensitive to these effects.

Although the inverse square law has been measured many times over the last two and a half centuries [3, 7–11], this subject has seen little progress in the last three decades. The smallest laboratory-based limit on the photon rest mass was reported nearly 24 years ago [11]. In



Figure 1: (Color online) A cut-away cartoon of the proposed experiment. The diagram is not to scale, and some dimensions have been greatly exaggerated for visibility.

this experiment an alternating voltage was applied between two conducting shells, and the induced voltage between the outer shell and a third shell surrounding the other two was measured with solid-state electronics. This measurement improved upon the best previous measurement, 12 years old at the time [10], by only a factor of 2.5. In this paper we show that it should be possible to revitalize this key field of study and improve sensitivity by orders of magnitude using a new approach — charged particle matter-wave interferometry.

In the proposed experiment a possible Coulomb's-law violating electric field inside of a conducting shell is measured with an ion interferometer. As shown in Fig. 1, ions travel through a conducting cylinder nested inside of a second cylinder. The outer conductor is grounded, and a time-varying voltage is applied to the inner conductor. A beam of cold atoms passes through small holes in the conductors. The atoms are ionized with a laser beam, shown as an arrow in the figure, and pass through three gratings to form a Mach-Zehnder interferometer. If an electric field is present in the inner conductor, the two interferometer arms will pass through different potentials, resulting in a relative phase shift. Although physical gratings could be used [12], optical gratings would avoid serious problems including charge build-up and image charges induced in the gratings. With an appropriate choice of ion, Raman transitions could be used [13], making precise control of grating phases and state-selective readout of the interferometer phase possible.

To calculate the limit on the photon rest mass that could be achieved in such an apparatus, we start with a

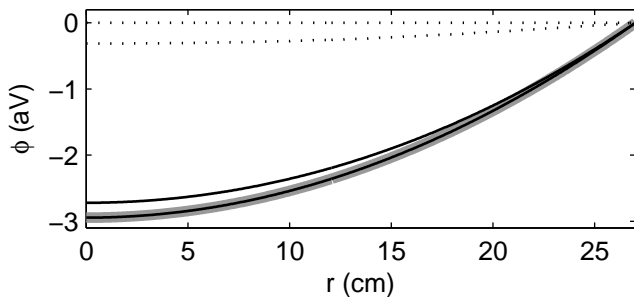


Figure 2: Calculations of potentials in a 2.6 m long, 27 cm radius tube held at 200 kV. The calculation assumes  $m_\gamma = 1 \times 10^{-50}$  grams. Potentials are plotted vs. the radial distance from the tube axis. The black lines are the deviation from the classical potential at axial distances of zero (lower line) and one meter (upper line) from the middle of the tube, plotted on top of a thick gray line representing the deviation for an infinite tube. The dotted lines show the calculated classical fringing-field potentials at the same locations multiplied by  $10^{35}$  to make them visible on this scale.

modified version of Laplace's equation:

$$\nabla^2 \phi - \mu_\gamma^2 \phi = 0. \quad (1)$$

This equation is derived from the modified version of Maxwell's equations generated by the Proca action for massive photons. In this equation  $\phi$  is the scalar electrostatic potential, and the small constant  $\mu_\gamma$  is related to the photon rest mass  $m_\gamma$  by the relation  $m_\gamma = \mu_\gamma \hbar / c$  where  $\hbar$  is Planck's constant divided by  $2\pi$ , and  $c$  is the canonical speed of light in vacuum.

In the limit as  $\mu_\gamma \rightarrow 0$ , Eq. 1 becomes Laplace's equation. For the simple case of a spherically symmetric system, Laplace's equation has the familiar solutions  $\phi(r) = A/r$  and  $\phi(r) = B$ , where  $A$  and  $B$  are constants. The  $A/r$  solution is the classical point-charge potential. The constant  $B$  solution allows us to arbitrarily define a point to be at zero potential without changing the fields described by the potential. If  $\mu_\gamma \neq 0$ , the solutions for a spherically symmetric system are a Yukawa potential  $\phi(r) = (A/r) \exp(-\mu_\gamma r)$ , and an exponentially growing solution  $\phi(r) = (B/r) \exp(\mu_\gamma r)$ . The Yukawa potential solution lets us interpret  $1/\mu_\gamma$  as an effective range of the Coulomb force. Without a constant solution, absolute potential has physical significance and we are no longer free to arbitrarily choose where  $\phi$  equals zero.

Due to the elongated geometry of the proposed experiment, we will approximate the finite inner conductor with an infinitely long tube. Numerical and analytical studies have verified that this is a good approximation for reasonably long tubes (see Fig. 2 and [14]). For a system with no angular or longitudinal dependence, solutions to Eq. 1 are the zeroth-order modified Bessel functions. Applying the conditions that  $\phi(r)$  must equal the applied voltage when  $r = R$  (the radius of the tube), and given the fact that  $\mu_\gamma$  is known to be small, we can approximate the po-

tential inside the inner tube with a lowest-order Taylor series in  $\mu_\gamma$ :

$$\phi(r) \approx (V + V_g) \left[ 1 + \frac{\mu_\gamma^2}{4} (r^2 - R^2) \right], \quad (2)$$

where  $V$  is the voltage applied to the inner tube relative to the outer tube, and  $V_g$  is the unknown voltage of the outer, grounded tube.

Rather than absolute potential, the interferometer will measure the potential difference between the two arms. Each of the arms in Fig. 1 consists of one horizontal and one diagonal segment. Both diagonal segments pass through identical potentials which induce equal phase shifts on the upper and lower arms. As such, the diagonal segments can be neglected. The horizontal segments, however, are at two different radii and travel through different potentials. Assuming that the two horizontal segments are a distance  $r_0$  and  $r_0 + s$  from the center of the tube, the potential difference between them is

$$\Delta\phi = \phi(r_0 + s) - \phi(r_0) \approx \frac{\mu_\gamma^2}{4} (V + V_g) (s^2 + 2r_0 s). \quad (3)$$

If  $\tau$  is the time that it takes the ions to travel the length of the horizontal segments, and  $e$  is the ion charge, the interferometer phase  $\Phi$  is given by

$$\Phi \approx \frac{e\mu_\gamma^2}{4} (V + V_g) (s^2 + 2r_0 s) \frac{\tau}{\hbar}. \quad (4)$$

Because we don't know  $V_g$ , and because many factors can offset the absolute phase of an interferometer, we would apply a time-varying potential  $V$  and look for a correlated change in the interferometer phase. For example, imagine that the applied potential  $V$  is periodically reversed. Because the  $\sim 700 \mu\text{F}$  capacitance of the Earth is very large compared to the  $\sim 1.6 \text{ nF}$  capacitance of the proposed conductors,  $V_g$  will remain roughly constant, and the difference in phase when  $V$  is reversed will be

$$\Delta\Phi \approx \frac{eV\mu_\gamma^2}{2} (s^2 + 2r_0 s) \frac{\tau}{\hbar}. \quad (5)$$

Solving Eq. 5 for  $\mu_\gamma$  we can determine the rest mass of the photon from the measured interferometer phase shift:

$$m_\gamma \approx \frac{\hbar}{c} \left[ \frac{2\hbar\Delta\Phi}{eV(s^2 + 2r_0 s)\tau} \right]^{1/2}. \quad (6)$$

To estimate the smallest detectable  $m_\gamma$ , it is useful to rewrite Eq. 6 in terms of the parameters which can be directly set experimentally. One important parameter is the velocity of the ions  $v$ . A smaller velocity results in larger diffraction angles. But if  $v$  is too small, stray electric fields can have a significant effect on ion velocities and trajectories. These fields will be extremely small inside the tube at the locations of the gratings (see Fig.

2), but they could be much larger in the region where the ions are generated. We can write the velocity as  $v = (2eV_s/m)^{1/2}$  where  $m$  is the mass of the ions and  $V_s$  is the voltage which would just bring the ions to a stop. Then we can set  $V_s$  to be several times the level of the expected stray fields to be sure that the trajectory of the ions is not greatly perturbed by them.

Two other important parameters are the maximum excursion of the ions from the center of the tube,  $a = r_0 + s$ , and the distance between gratings,  $L$ . A larger tube radius accommodates a larger separation  $s$  and offset  $r_0$ . A larger grating separation  $L$  means that the ions will interact with the field longer ( $\tau = L/v$ ) and results in a greater separation of the two arms of the interferometer ( $s \approx Lh/mvd$ , where  $h$  is Planck's constant and  $d$  is the grating period). With these parameters in mind, we can rewrite Eq. 6 as

$$m_\gamma \approx \frac{\hbar}{cL} \left[ \frac{\Delta\Phi dV_s}{\pi V a (1 - Q)} \right]^{1/2} \quad (7)$$

where the parameter  $Q = s/2a = \pi L\hbar/(2emV_s a^2 d^2)^{1/2}$  shows very weak dependence on ion mass and charge — although higher charge and lower mass results in more precision for a given ion velocity, this is offset by the greater velocity needed to overcome deflections by stray fields. For arbitrary experimental parameters,  $0 < Q \leq 1/2$ . For the parameters selected below,  $Q$  is small for all possible ion masses, ranging from  $1 \times 10^{-2}$  for  $^1\text{H}^+$  to  $1 \times 10^{-3}$  for  $^{133}\text{Cs}^+$ , and the precision of the experiment will not change much with the mass or charge of the ion.

There are practical limits on  $L$  and  $a$  for a table-top apparatus. We chose  $L$  to be one meter, and limited  $a$  to be a conservative 25 cm. For our numerical calculations (Fig. 2) we assumed a total length for the inner tube plus end-caps of 3 m, and a tube radius of 27 cm. This gives sufficient space to limit fringing fields, to be sure that the infinite-tube calculation is a good approximation in the region of the interferometer, and keeps the ions in the interferometer from traveling close to the tube surface. Only  $a$ , and not the outer radius of the tube, affect the precision predicted in Eq. 7. So a tube with a larger radius could be used to further limit ion-surface interactions without changing the predicted precision.

We assumed a grating period of 200 nm, about half the wavelength of a readily-available uv diode laser. We selected a value of 200 kV for  $V$  because it is within the range of what is possible with off-the-shelf power supplies and vacuum feed-throughs. Higher voltages could possibly be obtained by using the two tubes as a capacitor in a resonant circuit. Based on work done with atom interferometers [15], it should be possible to detect phase-shifts as small as  $10^{-4}$  radians. We set the final parameter,  $V_s$ , to 0.5 mV assuming that voltages due to stray fields can be controlled well below this level.

With these parameters we predict a sensitivity to photon rest mass of  $9 \times 10^{-50}$  grams, nearly two orders of magnitude smaller than the limit reported in [11]. The

largest errors are expected to be due to inertial-force shifts [15] and ion-ion interactions. The ion-ion interactions can be reduced by limiting the number of ions inside the conductor at any given time — in the limit of a single ion at a time, this effect disappears while still affording a count rate of tens to hundreds of ions per second. This drift can also be reduced by using non-classical, anti-bunched ion beams.

We performed numerical calculations to verify that fringing fields from holes in the conductor could be made negligibly small [14]. In these calculations the inner conductor was capped with 20 cm long end caps to reduce fringing fields. Because the calculation assumed axial symmetry, the effect of the holes in the end cap were simulated with ring-shaped apertures. As such, the calculation greatly overestimates the size of the fringing fields. The results of the calculations were verified with piecewise analytical solutions to the conductor geometry. The calculations show that fringing fields will be tens of orders of magnitude below the detection limit (see Fig. 2), and should not be an issue. Any small drifts in patch charges or similar effects should not be correlated with the applied voltage difference, especially considering the extremely small level of the fringing fields. As such, these drifts could be averaged out if they are not too large. The susceptibility of the proposed experiment to stray electric fields should be no greater than in other recent laboratory tests of Coulomb's law, and in [11] it was implied that these effects were not a limitation.

Care will have to be taken to magnetically shield the inner conductor (possibly by using a type-I superconductor for the outer conductor). In addition, the magnetic fields created by the charging and discharging of the conductors will have to be taken into account. The effect of these time-varying fields could be removed by calculating what the fields would be or by measuring them directly, or they could be reduced exponentially by increasing the time between voltage reversals. Also, in addition to small changes in electric and magnetic fields, the presence of larger static fields could also significantly reduce contrast in the interferometer. Nevertheless, it would seem that these difficulties are surmountable in a reasonable experiment.

Given the assumed parameters the separation  $s$  would range from 6.4 mm (for  $^1\text{H}^+$ ) to 0.56 mm (for  $^{133}\text{Cs}^+$ ), and the ion beam would enter the apparatus at a radius  $r_0$  ranging from 24.4 cm ( $^1\text{H}^+$ ) to 24.9 cm ( $^{133}\text{Cs}^+$ ). These parameters yield an  $s$  for electrons which is greater than the radius of the tube, so for these experimental parameters electrons are not an ideal choice. For a horizontal apparatus in gravity, assuming that the ions undergo a parabolic trajectory with the peak at the location of the center grating, the ions will fall a vertical distance ranging from 51  $\mu\text{m}$  ( $^1\text{H}^+$ ) to 6.8 mm ( $^{133}\text{Cs}^+$ ), giving them a vertical velocity of only  $1.0 \times 10^{-4}$  ( $^1\text{H}^+$ ) to  $1.4 \times 10^{-2}$  ( $^{133}\text{Cs}^+$ ) times their longitudinal velocity. The phase shift due to gravity will be constant as the applied voltage is reversed, and will not affect the measurement.

Obtaining a precision of  $9 \times 10^{-50}$  grams requires the use of slow ions. These ions could be generated from a slow neutral atom beam. The velocity, determined by  $V_s$  and the mass of the ion, ranges from 311 m/s ( $^1\text{H}^+$ ) to 27 m/s ( $^{133}\text{Cs}^+$ ). The latter is a reasonable velocity for a beam of atoms from an LVIS source [16]. Higher velocities are easily obtained by accelerating the ions with a small potential. As such, any atom that can be laser cooled could be used to obtain the above stated sensitivity. However, lighter ions have the advantage of faster transit, which would make it possible to modulate the voltage applied to the tube at a higher frequency, reducing the effective bandwidth of systematic drifts.

The predicted precision is considerably better than that reported previously for laboratory experiments. Furthermore, a potential pitfall is avoided. In the most recent experiments [10, 11] a voltage between two conducting shells was measured electronically. Any non-zero electric field would tend to draw a charge through the electronics to cancel the field. If 0.001 times the charge of an electron passed through the probe electronics, it would cancel the field due to a photon mass larger than the reported precision. In our scheme the only influence the ions have on the system under test is the well-understood induction of an image charge in the conductor.

In addition to limits on the photon rest mass, following the tradition of Cavendish [8] it is also common to parametrize inverse-square law violations with a small parameter  $\delta$  by assuming that the potential of a point charge falls off as  $r^{-(1+\delta)}$ . Because the Proca treatment isn't necessarily correct, this additional figure of merit is valuable. Unfortunately the  $r^{-(1+\delta)}$  potential does not come from an underlying theory. If such a theory existed,  $r^{-(1+\delta)}$  would enter naturally as a solution to a modified

version of Laplace's equation. At least one more solution, one which is finite at  $r = 0$ , must exist. Knowing just one of the solutions is not sufficient to determine charge distributions.

It appears that previous experiments calculated limits on  $\delta$  by integrating the point-charge potential over the classical charge distribution. But the unmodelled deviation of the true charge distribution from the classical distribution could have a big impact on the implied magnitude of  $\delta$ . Furthermore, if the unknown equation is nonlinear, the potential cannot be related to an integral of point charges. It is also somewhat disturbing that in this formalism the units of the permittivity  $\epsilon_0$  depend on  $\delta$ . Ignoring these concerns, we integrated the point-charge potential over the classical charge distribution and predict that a limit on  $\delta$  of a few times  $10^{-22}$  would be possible in the proposed apparatus. This would represent an improvement of five orders of magnitude over the value of  $6 \times 10^{-17}$  reported in [11].

In conclusion, we have discussed the potential of ion interferometry in searches for violations of Coulomb's law. Calculations using reasonable parameters suggest that a table-top device should be able to measure photon rest mass at the level of  $9 \times 10^{-50}$  grams, and measure deviation in the exponent of Coulomb's inverse square law at the level of a few times  $10^{-22}$ , both representing an improvement of several orders of magnitude over current laboratory measurements. In addition, the apparatus would be immune to effects related to the modification of the field by the instrument used to measure it.

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