Allowed: One sheet of notes, pencils, scratch paper, calculator, ruler. Foreign students may also use a Foreign to English dictionary (only testing center dictionaries, no outside dictionaries) and/or an English dictionary (only testing center dictionaries, no outside dictionaries). No time limit.

Each multiple choice problem is worth 4 points. Unless otherwise indicated, for multiple choice questions with numerical answers, the choices represent the answer rounded to three significant figures. Answers to the multiple choice problems should be recorded on your bubble sheet.

Each of the worked problems is worth 9 points. Your work and answers to the worked problems should be done on your “Worked Problem Answer Sheet.” Put your correct CID on the Worked Problem Answer Sheet or you will lose 5 points! If you choose to keep your note sheet or your scratch paper, make sure they are stapled well and aligned with the answer sheet, or you will lose up to 5 points!

To receive full credit on the worked problems section, please show all work clearly and write neatly. If you wish to get partial credit on problems with incorrect answers, be sure to solve all questions algebraically first, then plug in numbers (with units) to get the final answer. Unless otherwise instructed, give all numerical answers to three significant digits in SI units.

Do not do work for one problem in space allotted for another problem.

Remember to keep extra digits in intermediate results, otherwise your final answer may be off. Also, check that your results make sense and have correct units. You can also consider what happens when you set a variable to a value that gives you a known result.

Possibly Useful Information

Near Point of a Healthy Eye = 25.0 cm

Multiple Choice

1. From the vantage point of Earth, two particular stars are $9.88 \times 10^{-5}$ degrees apart. What is the minimum diameter that the primary mirror in a telescope must have for me to be able to resolve the two stars according to Rayleigh’s criterion? Assume a wavelength of light of 570 nm.

(a) $0.04 \times 10^{-7}$ m
(b) $4.03 \times 10^{8}$ m
(c) $7.04 \times 10^{-3}$ m
(d) $4.03$ m
(e) $7.04 \times 10^{-2}$ m
(f) $0.403$ m
(g) None of the above

2. A nearsighted person has a far point of 306 cm. What is the focal length of the lenses they need in their glasses?

(a) $-23.1$ cm
(b) $-306$ cm
(c) $-27.2$ cm
(d) $23.1$ cm
(e) $-612$ cm
(f) $306$ cm
(g) None of the above

3. I place a curved mirror on the ground. I look into it, and all of the light from the sun which reflects off of the mirror comes to a focus on my nose, a distance of 27.0 cm above the mirror, causing an instant of pain and a puff of smoke. What is the radius of curvature of the mirror?

(a) $7.41 \times 10^{-2}$ cm
4. A laser beam travels from a medium with an index of refraction of 1.53 into a second medium with an index of refraction of 1.19. If the angle that the laser beam makes, relative to the normal of the interface, in the first medium is $\theta_1$, and the angle relative to the normal in the second medium is $\theta_2$, which of the following is true?

(a) $\theta_2 < \theta_1$
(b) $\theta_1 = \theta_2$
(c) $\theta_1 < \theta_2$

5. An object is placed 73.4 cm away from a convex mirror with a focal length of -82.9 cm. Which of the following will be true of the image formed of the object?

(a) It will be real and upright, and it will be on the opposite side of the mirror as the object
(b) It will be virtual and inverted, and it will be on the same side of the mirror as the object
(c) It will be virtual and upright, and it will be on the same side of the mirror as the object
(d) It will be real and inverted, and it will be on the opposite side of the mirror as the object
(e) **It will be virtual and upright, and it will be on the opposite side of the mirror as the object**
(f) It will be real and upright, and it will be on the same side of the mirror as the object
(g) It will be real and inverted, and it will be on the same side of the mirror as the object
(h) It will be virtual and inverted, and it will be on the opposite side of the mirror as the object

6. A thin lens has a front surface which is convex with a radius of curvature of 8.53 cm, and a back surface which is convex with a radius of curvature of 8.87 cm. If the index of refraction of the glass is 1.44, what is the focal length of the lens $f$?

(a) $-1.98 \times 10^{-3}$ cm
(b) 506 cm
(c) $-9.88$ cm
(d) 0.101 cm
(e) $1.98 \times 10^{-3}$ cm

(f) $9.88$ cm
(g) None of the above

7. I place a book a distance of 30.6 cm away from a lens with a focal length of 52.7 cm. What is $q$ for the image of the book?

(a) $5.27 \times 10^{-3}$ cm
(b) 22.1 cm
(c) 190 cm
(d) 73.0 cm
(e) $-1.37 \times 10^{-2}$ cm
(f) $730$ cm
(g) 19.4 cm
(h) None of the above

8. A ray of light passes from a material with an index of refraction of 1.52 into a second material with an index of refraction of 1.16. I measure the angle between the direction that the ray of light travels in the first material and a line which is normal to the surface between the two materials, and I find it to be $14.8$ degrees. What is the angle between the ray of light in the second material and the normal of the surface?

(a) $0.295$ degrees
(b) **19.6 degrees**
(c) $0.645$ degrees
(d) $11.2$ degrees
(e) $9.66$ degrees
(f) There will be no ray of light in the second material
(g) None of the above

9. I shoot a laser beam with a wavelength of 711 nm at a piece of foil with two tiny slits, a distance of 621 micrometers apart. This results in an interference pattern on a wall a distance 3.99 meters away. How far apart are the centers of the bright fringes in the center of the interference pattern?

(a) $7.11 \times 10^{-7}$ m
(b) $3.48 \times 10^{3}$ m
(c) $6.21 \times 10^{-4}$ m
(d) $1.14 \times 10^{-3}$ m
(e) **4.57 \times 10^{-3} m**
(f) $7.77 \times 10^{-3}$ m
(g) None of the above

10. I place a flower a distance of 32.6 cm away from a lens with a focal length of $-92.8$ cm. Which of the following will be true of the image that the lens makes of the flower?
(a) It will be inverted, real and on the opposite side of the lens from the object.
(b) It will be inverted, real and on the same side of the lens as the object.
(c) It will be inverted, virtual and on the opposite side of the lens from the object.
(d) It will be inverted, virtual and on the same side of the lens as the object.
(e) It will be upright, real and on the opposite side of the lens from the object.
(f) It will be upright, real and on the same side of the lens as the object.
(g) It will be upright, virtual and on the opposite side of the lens from the object.
(h) It will be upright, virtual and on the same side of the lens as the object.

11. It turns out that the diffraction pattern you get when you block a laser beam with an object of thickness \( a \) is basically the same as what you get when you allow the laser to pass through a slit of width \( a \). Imagine I place a hair in a laser beam with a wavelength of 709 nm. The dark strips on either side of the central bright fringe in the resulting diffraction pattern on a wall a distance of 14.8 m away are separated by a distance of 142 mm. How thick is the hair?

(a) 296 \( \mu \)m
(b) 0.843 \( \mu \)m
(c) 148 \( \mu \)m
(d) 18.5 \( \mu \)m
(e) 37.0 \( \mu \)m
(f) 73.9 \( \mu \)m
(g) None of the above

12. A ray of light is propagating inside a piece of glass with an index of refraction of 1.34. The ray strikes the edge of the glass from the inside, at an angle of \( \theta \) relative to the normal of the surface. For what values of \( \theta \) will all of the light reflect off the surface (with no light transmitted out of the glass into the air)?

(a) \( \theta < 0.842^\circ \)
(b) \( \theta > 0.842^\circ \)
(c) \( \theta > 55.8^\circ \)
(d) \( \theta < 48.3^\circ \)
(e) \( \theta > 48.3^\circ \)
(f) \( \theta < 55.8^\circ \)
(g) There are no angles which result in no transmitted light

13. I look through a telescope at a carrot a distance of 479 meters away from me. The carrot is 10.9 cm tall, and the telescope has an objective focal length of 233 cm and an eyepiece focal length of 1.59 cm. What is the magnitude of the angular size of the image of the carrot you are looking at?

(a) 1.91 degrees
(b) \( 2.02 \times 10^6 \) degrees
(c) 4.83 degrees
(d) \( 8.90 \times 10^{-5} \) degrees
(e) \( 1.55 \times 10^{-6} \) degrees
(f) \( 3.33 \times 10^{-2} \) degrees
(g) None of the above

14. I'm trying to read the text on a piece of microfilm with a microscope. I can only read words if the letters are at least 0.5 mm tall, but the letters on the microfilm are \( 2.50 \times 10^{-4} \) mm tall. The microscope has an eyepiece with a focal length of 6.68 mm. The lenses are 25.7 cm apart. What should the focal length of the objective be if I want the image of the text to have the same angular size as a document with 0.5 mm tall letters?

(a) 308 mm
(b) 4.81 mm
(c) 9.62 mm
(d) 616 mm
(e) \( 6.87 \times 10^{-3} \) mm
(f) 0.208 mm
(g) None of the above

15. A piece of metal is cut with an end mill, leaving the surface with a regular pattern of scratches. You shine a laser with a wavelength of 726 nm at the surface at normal incidence, and you note that the first diffraction order is at an angle of 20.1° relative to the normal of the surface. How far apart are the scratches?

(a) \( 1.06 \times 10^3 \) nm
(b) 765 nm
(c) \( 2.11 \times 10^4 \) nm
(d) 249 nm
(e) 125 nm
(f) 689 nm
(g) None of the above
16. With the sun nearly directly behind your head, you look at a soap bubble in front of you. Using a diffraction grating, you determine that there is a reflection maximum for light with a wavelength of 649 nm at the center of the bubble, where light from the sun strikes it at normal incidence. Which of the following is a possible thickness for the bubble wall at that location? Assume that the bubble wall has an index of refraction of 1.32.

(a) 974 nm  
(b) $1.48 \times 10^3$ nm  
(c) $1.72 \times 10^3$ nm  
(d) $1.14 \times 10^3$ nm  
(e) 860 nm  
(f) 738 nm  
(g) None of the above

**Free Response**

17. We want to minimize the reflections off of a window by placing a thin film on it. If the window is made of a material with an index of refraction of 1.30, and the film is made of something with an index of refraction of 1.54, what is the smallest nonzero thickness of film that will result in a minimum reflection at a wavelength of 688 nm?

18. An object is placed a distance of 4 cm from a mirror with a focal length of -4 cm, shown as an arrow and a vertical line on your Worked Problem Answer Sheet. Draw at least three rays on the sheet to determine the size and location of the image formed. Then calculate the image distance $q$ and the lateral magnification $M$. Finally, state whether the image is real or virtual. If you didn’t bring a ruler, you may fold a piece of scratch paper to use as a straight edge.

19. An astronomer sends light from a distant star through a diffraction grating. She notices that one of the wavelengths of light from the star makes a first order diffraction maximum at an angle of 69.9 degrees. If the diffraction grating has $1.68 \times 10^3$ lines per millimeter, what is the wavelength of the light making the bright diffraction fringe?

20. An object is placed a distance of 5 cm from a lens with a focal length of -9 cm, shown as an arrow and a vertical line on your Worked Problem Answer Sheet. Draw at least three rays on the sheet to determine the size and location of the image formed. Then calculate the image distance $q$ and the lateral magnification $M$. Finally, state whether the image is real or virtual. If you didn’t bring a ruler, you may fold a piece of scratch paper to use as a straight edge.
Solutions

17. Since $n_{\text{film}} > n_{\text{obj}}$, we get a $180^\circ$ phase shift on the first reflection, but not on the second one. So the two reflections are already out of phase, and we don’t want to change that. So we want the path through the film to be an integer number of wavelengths in the film. The light passes twice through the film, so the path through the film is $2t$. And the wavelength in the film is $\lambda/n$. So

$$2t = \frac{m\lambda}{n}$$

where $m$ is an integer. The smallest nonzero thickness is when $m = 1$, so

$$t = \frac{\lambda}{2n_{\text{film}}} = \frac{688 \text{ nm}}{2 \cdot 1.54} = 223 \text{ nm}$$

18.

$$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{-4.00} - \frac{1}{4.00}\right)^{-1} = -2.00$$

$$\frac{q}{p} = \frac{-2.00}{4.00} = -0.500$$

The image is virtual.

19. The $m^{th}$ order bright fringe occurs at an angle which obeys the equation

$$d \sin(\theta) = m\lambda.$$ 

Solving this for $\lambda$, we get

$$\lambda = \frac{d}{m} \sin(\theta).$$

The order $m$ is one, and $d$ is one millimeter divided by $1.68 \times 10^3$. So

$$\lambda = \frac{1.0 \times 10^{-3} \text{m}}{1.68 \times 10^3} \sin(1.22) = 559 \text{nm}$$

20.

$$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{-9.00} - \frac{1}{5.00}\right)^{-1} = -3.21$$

5
The image is virtual.