

- (7 pts) An object with mass $m = 1$ kg is moving with a velocity which has an x component $u_x = -0.4 c$, a y component $u_y = 0.5 c$ and a z component $u_z = 0$. (a) What are the three components of the object's momentum? (b) What is the kinetic energy of the particle? (c) What is the total energy of the object (including its rest energy)? Give all answers in S.I. units.
- (8 pts) Now consider the same object as seen by an observer in a reference frame moving at a velocity $v = 0.7 c$ in the x direction relative to the frame used in problem 1. (a) Find the three components of the object's momentum and the total energy (including the rest energy) in the new frame by first transforming the three components of the particle's velocity and then plugging these transformed values into the same equations you used above. (b) Now use the momentum/energy transformation equations below and show that you get the same values:

$$\begin{aligned} p'_x &= \gamma(p_x - vE/c^2) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - vp_x) \end{aligned}$$

- (8 pts) I place a perfect clock which never loses or gains time and has a display which reads out to 1/100,000,000 of a second on top of a 100 meter tall building. I place an identical clock on the ground. Both clocks are synchronized at time $t=0$. (a) After ten years, a physicist named Liz standing next to the lower clock compares the times on the two clocks. If she assumes that the lower clock is correct, will she conclude that the upper clock is running fast or running slow? (b) By how much will she perceive the upper clock's time to differ from the lower clock's time? (c) If a physicist named Bill is standing next to the upper clock, will he conclude that the lower clock is running fast or slow? (d) By how much will Bill perceive the lower clock's time to differ from the upper clock's time? Assume that the acceleration due to gravity near the Earth is 9.80 m/s^2 and that the observers take into account and subtract the time that it takes for light from the faces of the clocks to reach their eyes when they are measuring the time. *Remember, relativity is NOT about things seeming to be delayed because it takes light time to reach your eye. This is a separate effect. Remember my illustration of a whole team of people making measurements directly in front of them...*
- (7 pts) (a) If I compress a 1 kg brick into a non-rotating black hole, what will be the radius of its event horizon (also known as the Schwarzschild radius)? (b) Classically an object undergoing a circular orbit of radius r with a velocity v undergoes an inward radial acceleration equal to $a = v^2/r$. Since light travels at the speed of light ($v = c$), what does r need to be for a photon of light to make a circular orbit around an infinitesimally big speck with a mass of 1 kg? The gravitational constant $G = 6.6726 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, and the force from a point mass of mass M on a second point of mass m a distance r away is given by $F = mMG/r^2$.

Extra problems I recommend you work (not to be turned in)

- In class we derived an equation for the gravitational redshift assuming the acceleration due to gravity is constant. This works well when we are comparing two clocks near the surface of the Earth, but not for clocks far away from the Earth. Derive an equation for the fractional shift in the frequency of two clocks ($\Delta f/f$) if the two clocks are a distance r_1 and r_2 from the center of a spherically symmetric planet with a mass m . Assume both clocks are above the surface of the planet.
- Imagine a photon is created a distance r away from the 1 kg infinitesimal speck and travels radially away from the speck. As it travels away it gets red-shifted. How big must r be in order for the photon to not get red-shifted down to zero frequency before it gets infinitely far away from the speck?
- Use the velocity transformation equations to derive the energy/momentum transform equations.

- (a) What makes up the missing mass of the universe? (b) What is the fundamental mechanism which causes the collapse of a quantum wave function? (c) What general method can be used to combine relativity with quantum mechanics? (d) Why is the universe made of matter instead of antimatter? (e) What is the analytical form of the nuclear weak force? (f) Can CPT be violated? ... Aaaahhh! So many unanswered questions! This may be the last homework, but there are still so many problems to solve! Why are you still reading this... get busy! There are great discoveries to be made!!!