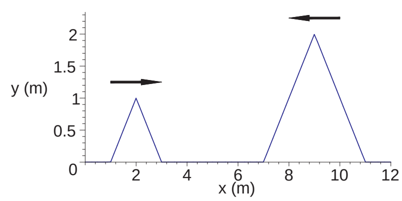
Physics 123 section 2 CID ____________

Homework 14

Score ________ / 30

1. (3 pts) You are abducted by aliens and placed in a holding cell on an unknown planet. Due to your diligent study of the Starfleet Planetary Guide, you know that if you could determine g, the gravitational acceleration on the planet, you would be able to figure out where you are. So you pull a thread from your uniform which is 1.21 meters long and which weighs 0.402 grams. You tie the end to your shoe, which weighs 0.211 kg. You then hold the top of the string with the shoe hanging at the bottom, and you pluck the string near the top. The pulse takes 0.312 seconds to travel down to the shoe. (a) What is g? (b) If I hang *both* shoes, rather than just one shoe from the string, thereby doubling the tension in the string, by what factor with the speed of waves on the string increase?

2. (4 pts) Two triangular shaped pulses are traveling down a string, as shown in the figure below. The figure represents the state of the string at time $t = 0$. The pulse on the left is traveling to the right, and the pulse on the right is traveling to the left, as indicated by the arrows above the pulses in the figure. The speed of waves on the string is 1 m/s. Draw the shape of the string at the following times: $t = 2$s, $t = 2.5$s, $t = 3.5$s, and $t = 5$s.



3. (6 pts) Imagine your slinky stretched to a length L and fixed at both ends.

(a) What is the tension T and the linear mass density μ of the slinky in terms of its mass m, its spring constant k, and its length L? Assume that the stretched length of the slinky is long enough compared to the length when it is not stretched that the unstretched length is negligible.

(b) What is the wave speed for transverse waves on a slinky (in terms of m, k, and L)?

(c) Have someone hold one end of your slinky (or attach it to something like a doorknob). Take the other end and stretch the slinky until it is about five feet long. Now strike one end of the slinky to make a transverse pulse and watch as the pulse travels to the other end and then reflects back. Time how long it takes for the pulse go out and back 10 times, and use this to calculate the wave speed for transverse waves on the slinky.

(d) Now predict what the wave speed would be if the slinky were stretched to about 10 feet.

(e) Stretch the slinky until it is about 10 feet long and measure the wave speed the same way you did before.

4. (2 pts) A transverse pulse travels down a slinky and reflects off of the end which is being held by a friend of yours. (a) Will the reflected pulse look the same as the incoming pulse, or will it be inverted? Now have someone hold one end of your slinky (or attach it to something like a door knob). Take the other end and pull it back until the slinky is stretched about 10 feet (don't stretch it too far or it won't slink back together again and the slinky will be ruined). Hold your end of the slinky at arms length so that you can see the slinky well. Now quickly strike the top of the slinky with your hand to make a transverse pulse. Watch as the pulse reflects off of the fixed end. (b) Was the reflected pulse inverted?

5. (5 pts) If a and b are real numbers and $\tilde{c}$ is a complex numbers, what is the complex conjugate of (a) $a + b$, (b) $a + ib$, (c) $a + \tilde{c}$, (d) $a + i\tilde{c}$, and (e) $(a + ib)/[\sin(\tilde{c}) + ia]$?

6. (2 pts) Show that $\tilde{A}^* \tilde{A} = |\tilde{A}|^2$.

7. (5 pts) I want you to know how to derive the rate at which a sine wave transmits power down a string. The basis idea is this — every cycle the sum of the kinetic and potential energy contained in a piece of string of length λ passes by any given point on the string. So imagine a long string with a linear mass density μ under a tension T with a sine wave of the form $y(x,t) = A\sin(kx - \omega t)$ traveling on it. Now consider an infinitesimal piece of the string with an unstretched length (its length when no wave is passing through it) of dx. This piece of string will have a mass $dm = \mu dx$. When the wave pass through it, it moves at a velocity dy/dx, giving it some kinetic energy. In addition, its length stretches, giving it potential energy. The x component of its length is still dx, but it now has a y component of length, such that its total length ds is

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

We will assume that the amplitude of the wave is small such that $dy \ll dx$. This allows us to use a Taylor series and make the approximation:

$$ds = dx\left(1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right).$$

It turns out that the amount of potential energy is exactly equal to the amount of kinetic energy. So to make your life simpler, I'm going to let you choose to find *either* the kinetic or potential energy, not both. So choose one version of problem 7 below and work it. Assume that the amplitude of the wave is small enough that T is essentially constant as the wave passes by, and give all of your answers in terms of A, μ, ω, and k. For either one, you will need the following integral:

$$\int_a^{a+\lambda} \cos^2\left(\frac{2\pi x}{\lambda}\right) dx = \int_a^{a+\lambda} \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{\lambda}{2}.$$

Kinetic: (a) What is the kinetic energy dE_k that this infinitesimal piece of string contains at time $t = 0$? (b) Now consider a finite piece of the string with an unstretched length one wavelength long. How much kinetic energy does this piece of string have at time $t = 0$?

Potential: (a) Remembering that work equals force time distance, what is the potential energy dU contained in an infinitesimal piece of the string at location x at time $t = 0$? Hint - how much work would you have to do to stretch this piece string from its original length dx to its length when the wave is passing through, assuming that the tension in the string doesn't change significantly as we do this. (b) What is the total potential energy in one wavelength of the string?

8. (3 pts) We know that the the total energy in one wavelength (kinetic plus potential) will pass a point on the string once per period T. Use this and the formulas you found above to prove that

$$P = \frac{1}{2}\mu\omega^2 A^2 v.$$

<u>Extra problems I recommend you work (not to be turned in)</u>

- Hold one end of your slinky up high and let the other end dangle downward (don't let it touch the floor). (a) If you whack the end of the slinky to make a transverse pulse, what will happen to the pulse when it reaches the bottom? Will it reflect? Will the reflection be inverted? (b) Try it and see what happens. You may be surprised! (c) Does the dangling end of the slinky act as a free or a fixed end? Why?
- If you hold one end of your slinky up high and let the other end dangle down (without touching the floor), how will the wave speed change as a function of the distance from the bottom of the slinky?
- Use your answers from the above question and from problem 4 to predict the time it would take for a transverse pulse to travel down and back up the slinky 5 times. Now test your prediction.
- Find the other component of the energy in a wave (the one you chose not to work).
- In our derivation for power transmitted by a sine wave we integrated over a wave. As such, what we really found is the "average" power transmitted through the string. It turns out that the power passing by a point goes up and down with time. Find an equation for the instantaneous power passing a point x.