1. (2 pts) Explain why any possible reversible engine operating between two thermal reservoirs must have exactly the same efficiency as a Carnot engine.

2. (4 pts) The steam entering a steam engine is at a temperature of $T_h = 120\,^\circ C$. When it exits the engine it is at a temperature of $T_c = 105\,^\circ C$. (a) What is the maximum theoretical efficiency that the steam engine could have? (b) If the boiler injects 1.14 kW of power into the engine as heat, what is the maximum possible output power that the engine could produce?

3. (4 pts) I want to keep my freezer at a temperature of $-5\,^\circ C$ when my house is at $25\,^\circ C$. (a) What is the maximum possible coefficient of performance for a refrigerator operating between these two temperatures? (b) If 2 Joules of heat leak from the environment into my freezer each second, what is the minimum theoretical power that my freezer will consume to keep the temperature inside the freezer at $-5\,^\circ C$?

4. (6 pts) Consider the cycle we looked at in problem 3 on the last homework (if you don’t remember the answers that you got, please feel free to steal them from the solutions that I posted on Blackboard). Give your answers in terms of the same given quantities as you did for that problem. (a) If an engine were to be made using that cycle, what would its efficiency be (assume that it is an ideal engine with no friction, etc.)? (b) If you calculated the efficiency of a Carnot engine running between the minimum and maximum temperatures of the engine in part (a), would it be greater or less than the efficiency of the engine in part (a)? (c) Now actually calculate the efficiency of a Carnot engine running between these two temperatures.

5. (2 pts) The coefficient of performance (COP) for an ideal heat pump in heating mode is $Q_h/W = Q_h/(Q_h - Q_c)$. Because we know that $Q_h$ is always greater than $Q_c$, for any reasonable values of $Q_h$ and $Q_c$, the numerator in this equation will be greater than the denominator, resulting in a COP greater than one. Explain physically (not mathematically) why you would expect an ideal heat pump to always have a coefficient of performance greater than one. (Hint - an electric heater has a COP of one - all of the energy it consumes turns into heat in the room. What ultimately happens to the energy used to drive the heat pump?)

6. (8 pts) Lets derive the Carnot efficiency. Take a look at the PV diagram of the Carnot cycle below. Efficiency is defined to be $e = W/Q_{hot} = (Q_{hot} - Q_{cold})/Q_{hot}$. Unless otherwise noted, give all answers to this problem in terms of $n, T_{hot}, T_{cold}, V_A, V_B, V_C, V_D, \gamma,$ and fundamental constants.

(a) Find the heat that enters the gas during the adiabatic processes from B-C and from D-A. (In other words, what are $Q_{BC}$ and $Q_{DA}$?)

(b) Find the change in the internal energy of the gas change during the isothermal processes. (In other words, what are $\Delta E_{AB}$ and $\Delta E_{CD}$?)

(c) How much work is done on the gas during each isothermal process? (In other words, what are $W_{AB}$ and $W_{CD}$?)

(d) Use your results above to find $Q_{hot}$ and $Q_{cold}$.

(e) Use the adiabatic transitions to find a relationship between $(V_B/V_A)$ and $(V_C/V_D)$.

(f) Use what you found above to write the Carnot efficiency in terms of just $T_{hot}$ and $T_{cold}$.

7. (4 pts) Use what you learned in the last problem to derive the coefficient of performance for a Carnot heat pump in heating and in cooling mode.
Extra problems I recommend you work (not to be turned in)

- Compare the efficiency of an engine using an ideal Otto cycle with the efficiency of an ideal Carnot engine if both have the same minimum and maximum temperatures.

- Work some of the problems from your book on the Carnot cycle and refrigerators / heat pumps.