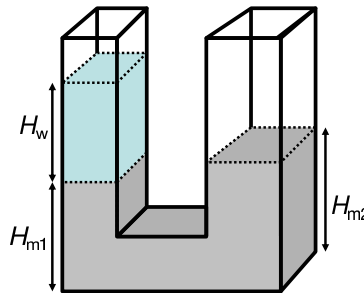


1. (4 pts) Consider the U-shaped glass vessel shown below. The tube rising up on the left has a cross-sectional area $A_1 = 4 \text{ cm}^2$. The tube on the right has a cross-section of $A_2 = 6 \text{ cm}^2$. Mercury is poured into the vessel, and then a small amount of water is poured into the left side of the vessel. The column of water is $H_w = 3.25 \text{ cm}$ tall. The top of the mercury on the left side is $H_{m1} = 3.01 \text{ cm}$ above the bottom of the vessel. What will the height H_{m2} of the top of the mercury column on the right side be?



2. (4 pts) (a) A thin spherical shell of Aluminum which is 2 meters in diameter contains helium gas at atmospheric pressure. How thick can the aluminum shell be if we want the sphere to float in air? (b) How thick can the aluminum be if the shell is filled with hydrogen rather than helium?
3. (3 pts) A pine board which is 2 meters long, 10 cm wide, and 2 cm thick is floating in a lake. How much of the 2 cm thickness will be above water?
4. (4 pts) Before applying the continuity equation and Bernoulli's equation to specific problems, let's examine where the equations come from. This will help you better understand how to use them, what approximations they assume, and how to come up with alternative equations when these approximations are not valid. Imagine that a tube has fluid flowing through it. The tube is bent such that one piece of the tube is at a height y_1 , and a second part of the tube is at a height y_2 . The fluid is flowing in the direction from point 1 to point 2. The cross-sectional area of the tube at the two points is A_1 and A_2 . Now imagine that at time $t = 0$, all of the fluid between point 1 and point 2 has red dye in it, to distinguish it from the rest of the fluid.
- (a) Imagine that during some short interval dt , one edge of the red fluid flows a distance dx_1 beyond point 1, and the other edge flows a distance dx_2 beyond point 2. Now imagine that *the fluid is incompressible*. If the fluid is incompressible, then the rate at which fluid flows past point 1 (in m^3/s) will be exactly the same as the rate it flows past point 2. Use this fact to find a relationship between dx_1 and dx_2 . Then divide both sides of the equation by dt to convert it into a relationship between v_1 and v_2 , the velocity of the fluid at points 1 and 2. This is the continuity equation.
- (b) Now let's get Bernoulli's equation by applying conservation of energy to the red fluid. After the edge red fluid has moved a distance dx_1 past point 1, its kinetic and potential energy will have changed. The net change in its energy must equal the work done on it. So the first step is to find the work done on it as it moves this distance. Let's assume that *the fluid has no viscosity*, so that the tube walls do not exert a component of force in the direction of the flow. This means that the only work done on the red fluid is the work done by the pressure of the un-dyed fluid on either side of the red fluid. Find the total work done by the surrounding fluid on the red fluid in terms of dx_1 , dx_2 , the pressure at point one P_1 , and the pressure at point two P_2 .
- (c) How much does the potential energy of the red fluid change as it moves this distance? (Hint, the motion of the fluid is equivalent to taking a tiny piece of the fluid near point 1, accelerating it to velocity v_2 , and putting it up near point 2).

- (d) Now let's assume *laminar flow with no turbulence*. This means that the only kinetic energy that the fluid has is its ordinary translational kinetic energy — there is no energy stored in swirling motion, etc., anywhere in the fluid. How much does the kinetic energy of the red fluid change as it moves this distance?
- (e) Add the change in kinetic to the change in potential energy and set it equal to the work done on the fluid. Now take all of the stuff related to point 1 to the left side of the equation, and all of the stuff related to point 2 to the right side of the equation and use the same ideas you used to derive the continuity equation to make this into Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

5. (3 pts) You place a nozzle on your garden hose to increase the speed of the water exiting the hose. The output end of the nozzle has an inner diameter of 0.241 cm. The garden hose has an inner diameter of 1.83 cm. (a) If the water comes out of the nozzle at a speed of 1.13 m/s, what is the velocity of the water in the hose? (b) The water coming out of the nozzle is at atmospheric pressure. If the Nozzle is 1.67 meters above the ground, what is the pressure of the water in a piece of the hose which is lying directly on the ground? Assume that the water behaves like an ideal fluid.
6. (4 pts) Water is pumped from a lake up to a village on top of a hill 325 meters above the lake. It is pumped through a pipe with a uniform cross section. The top of the lake and the air in the village are both at atmospheric pressure, so to get the water up to the village a pump is placed near the lake to increase the pressure in the pipe. (a) Assuming that the water behaves like an ideal fluid, what minimum gauge pressure must the pump produce at the bottom of the pipe in order to deliver an infinitesimally small trickle of water to the village? (b) How much must the pressure at the bottom of the pipe increase in order to deliver a flow rate of 0.1 m³/s? (c) How would your answers to (a) and (b) change if we included the effects of viscosity?
7. (4 pts) Imagine that you had a cylindrically shaped cup filled to a height h with water sitting on a level table top. If you poked a small hole in the side of the cup, water would shoot out in an arc and hit the table. (a) If you want to maximize the distance that the water goes before hitting the table, how far from the bottom of the cup should you poke the hole? (b) If I place my hole at this location, how far will the water travel before hitting the table? Assume that the hole is small enough that the height of the water in the cup doesn't change significantly over the time that you make the measurements, and assume that you can neglect viscosity.
8. (4 pts) *Now let's test this out:* Find a paper or Styrofoam cup — a cylindrical one would be best, but those are hard to come by. A water cup from the Cougar eat is good enough. Punch a small hole at the correct height to maximize the distance that the water will go before hitting the table. The hole should be small so that you can make your measurements before the height of the water in the cup changes appreciably, but not too small or viscosity will change your results. If you use a pencil to make your hole, you will probably do well, but you will have to watch what happens quickly before the water level in the cup drops. Now place your cup on the table and mark where you expect the water to hit the table. Put your finger over the hole, and fill the cup with water. Quickly remove your finger and note how close to your mark the water hits. (a) How close were you? (b) Now put tape over your hole and punch a new hole which is *higher* and try again. Did the water go further or not as far? (c) Now do the same with a hole *below* the optimum height. Did the water go further or not as far?

Extra problems I recommend you work (not to be turned in)

- An extremely precise scale is used to measure an iron weight. It is found that the mass of the weight is precisely 10 kg. If we suck all of the air out of the room and weigh the iron again, how much will our measurement change?
- Rework problem 6 but now assume that the cross section of the pipe is twice as large at the bottom than at the top.
- A lead weight is placed on one end of a cylindrical wooden log with a radius r . The combined mass of the log and the weight is m . The log is then placed in a fluid with a density ρ . Because of the weight, the log floats upright. (a) Show that if I push the log down from its equilibrium position, the log will undergo harmonic motion. (b) What will the period of the motion be? Use the letter g to represent the acceleration due to gravity. (Hints: You may need to review harmonic motion from Physics 121. To show that it will undergo harmonic motion, you simply have to show that the force on the log as a function of displacement from equilibrium has the same form as the force exerted by a spring when it is stretched or compressed from its equilibrium length.)