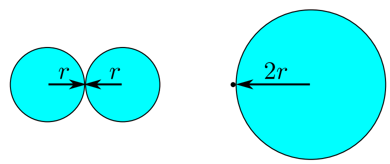
MEAN FREE PATH

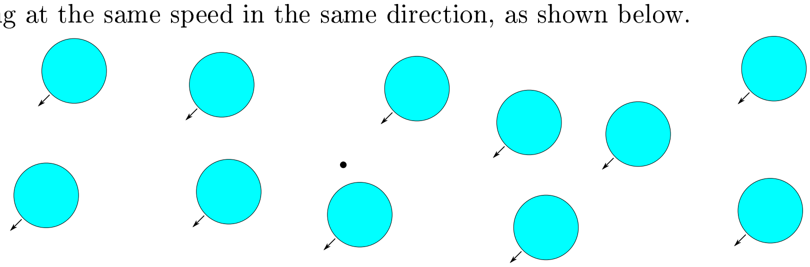
The mean free path, usually denoted as λ, is the average distance that a particle in a gas travels before bumping into another particle. It is important for a lot of different reasons. For example, if you do a gas-phase chemical reaction, reactions will only take place when molecules collide. So reactions can only take place at the mean collision frequency (which is just the average velocity divided by λ). In my research, we shoot atoms down a vacuum chamber and have them interact with laser beams. It's usually bad if the atoms we are studying bump into something. So we use the stuff I'm about to show you to figure out how low we need the pressure in the vacuum chamber to be to make sure that the mean free path is much longer than our experiment. Then we are safe.

Now that we see that the mean free path is important, it's time to calculate it. As you might expect, calculating what's going on in a mass of billions of billions of randomly moving molecules is daunting. So we're going to do a little hand waving. First of all, we are going to treat our molecules like hard spheres. If their centers get within a distance of $2r$ (where r is the radius of the spheres), a collision takes place - as shown in the figure on the left below. I repeat, *a distance from the particle centers of $2r$ means a collision happens.*

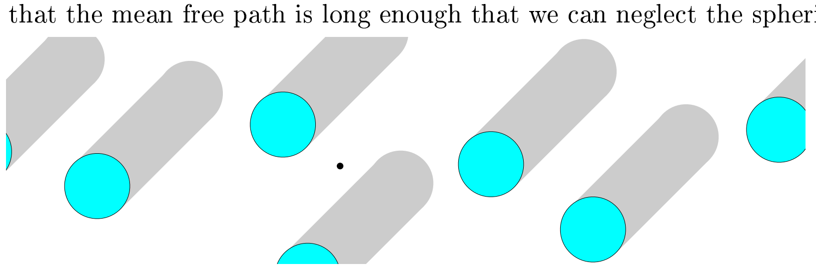


Because the magic distance is $2r$, we would get the same collision rate and the same mean free path for a test particle if instead of having a radius r the test particle had a radius of 0 and all of the other particles had a radius of $2r$. This is illustrated in the figure above on the right.

Now, to simplify matters further, we are going to pretend that all of the particles are standing still except for our test particle. This will underestimate the probability of colliding with stuff, but it will give as a good "order of magnitude" result. In the reference frame of our test particle, it is standing still, and all of the other particles are moving at the same speed in the same direction, as shown below.



As the molecules move past the test particle, they each sweep out a volume V_1 of where they have been. Each volume is a little tube with radius $2r$ and length l. (Technically, the ends of each tube will be spherical, but we will assume that the mean free path is long enough that we can neglect the spherical ends.)



Now we make another hand-waving argument: if the total volume swept out by each of the tubes is equal to the total volume of the gas, the chances that our test particle has collided with something are pretty good. So we'll assume that $NV_1 = V$ when $l = \lambda$. Here N is the number of molecules and V is the total volume of the gas. We solve this for λ to get the mean free path. We're almost done at this point. We just need to divide by $\sqrt{2}$ to make up for ignoring the fact that all of the molecules are moving. When we do this we get:

$$\lambda = \frac{1}{\sqrt{2} n_V \pi d^2},$$

where $n_V = N/V$ and $d = 2r$ is the diameter of the molecules (which is the radius of the tubes).