Physics 222 CID ___________

Homework 26

Score ________ / 30

Useful integral:

$$\int_0^\infty x e^{-(x/a)^2} dx = \frac{a^2}{2}$$

1. (7 pts) Cross sections in alternate universes.... Imagine that you had a piece of inzane which is made up of microscopic particles called spamons. If we irradiate our inzane with a beam of sillions, some of the sillions will be absorbed by the spamons, which will then release a giggle and become a hamon, according to the reaction: Sp(s, g)Ha. If a sillion passes within a distance r of a spamon, the probability of this reaction occurring is equal to $P(r) = e^{-(r/a)^2}$. In the following steps, we will calculate the cross section for this process.

- **(a)** Imagine a stream of sillions with a flux Φ which is streaming past a single spamon. Write a point on your paper to represent the spamon. Now draw a circle around the spamon, and then another circle just slightly larger than the first circle. We will call the area in between dA. In terms of dA and the flux Φ, what is the number of sillions per second which pass through this tiny piece of area.
- **(b)** We will call the radius of the inner circle r and the outer circle $r + dr$. Now in terms of Φ, r and dr, what is the number of sillions per second which pass through the tiny piece of area?
- **(c)** Each sillion that travels through this area has a probability of causing a reaction which is equal to $e^{-(r/a)^2}$. What is the rate dR at which sillions traveling through this tiny area cause this reaction to occur. Give your answer in terms of Φ, r, dr, and a.
- **(d)** Now integrate over all r to find the total rate R at which reactions with our lone spamon will occur as a function of Φ and a.
- **(e)** Even though the probability of absorbing a sillion is a continuous function of r, the reaction rate is the same as if all of the sillions striking a fixed area were absorbed, and none of the sillions passing outside this are absorbed. What is the "effective area" over which sillions are absorbed? This is the cross section σ.

2. (7 pts) Lets derive Beer's law! Imagine a cylinder with cross section A and length l with randomly placed nuclei with an average particle density (particles per unit volume) of n, each with a scattering cross section σ. Now imagine that a flux of neutrons Φ_0 enters one end of the tube. After traveling some distance x, the flux is a lower value $\Phi(x)$ because some of the neutrons have been absorbed. First consider a very thin slice of the tube of thickness dx. If it is thin enough, no cross sections of the nuclei in the slice will overlap, and the fraction of neutrons lost will be equal to the fraction of the tube's cross sectional area covered by the cross sections of the nuclei in the slice. (a) In terms of n, dx, and σ, what fraction of the neutrons will be lost as they traverse the slice? (b) Now use what you found in (a) to write down a differential equation of the form $d\Phi/dx =$something related to what you found in (a). (c) Now solve the differential equation to find $\Phi(x)$ in terms of Φ_0,l, σ, and n. Now you have Beer's law!

3. (4 pts) Assume that a particular reactor has a reproduction factor of $k = 1.7$. (a) How many generations does it take for the power level to double? (b) How many does it take to increase by a factor of 10? (c) If we ignore delayed neutrons, the time between generations in a uranium plant is about 1 ms. How long does it take for the power in a uranium reactor to go up by a factor of 10 if $k = 1.7$? Thank goodness for delayed neutrons!

4. (6 pts) The so-called "Coulomb barrier" is the energy needed to excite a ground state nucleus in order for it to have enough energy to split into two pieces (it is called the "Coulomb barrier" not because the Coulomb forces are what hold the nucleus together, but because this is the energy needed to pull the two pieces apart far enough that the Coulomb repulsion can overcome the nuclear binding). ^{239}U has a Coulomb barrier of 5.9 MeV and an atomic mass of $239.0542878u$. ^{236}U has a Coulomb barrier of 6.2 MeV and an atomic mass of $236.0455619u$. (a) How much kinetic energy must a neutron have to split a ^{238}U nucleus? (b) How much kinetic energy must a neutron have to split a ^{235}U nucleus? (c) How much kinetic energy must a proton have to split a ^{235}Pa? This isotope has an atomic mass of $235.0454368u$.

5. (6 pts) If I make a sphere of fissile material which is large enough it will spontaneously explode in a supercritical chain reaction (i.e. an atom bomb). But if the sphere is too small, too many neutrons will escape without causing another reaction, k will be less than 1, and I won't get an explosion. The mass of a sphere for which k is just equal to 1 is called the critical mass. Using declassified data, lets make a rough estimate of the critical mass of ^{235}U.

(a) The neutron capture cross section of ^{235}U for a neutron with a velocity typical of neutrons released by the fission of other ^{235}U is about 1.2 barns. The density of solid ^{235}U is 18.9 grams per cubic cm. About how far does a neutron travel through the uranium 235 before its probability of reacting with another ^{235}U reaches 50%?

(b) Since, on average, 2.4 neutrons are emitted per fission event, we need to make sure that the average neutron has a probability of being absorbed before leaving the sphere which is greater than $1/2.4$. To do this right, we need to integrate over possible angles that each neutron is emitted and over the possible places that each neutron may have originated (i.e., the volume of the sphere). But lets just wave our hands and say that the chances of absorbing at least 1 / 2.4 of the neutrons is pretty good if the diameter of our sphere is equal to the length calculated in (a). Calculate the mass of a sphere of ^{235}U with a radius equal to the half the distance you found in part (a).

(c) How much energy is released when the sphere in part (b) explodes? Assume that 200 MeV is released by each nucleus, and give your answer in units of tons of TNT (where 1 ton of TNT $= 2.62 \times 10^{22}$MeV, the amount of energy released when you set off one ton of dynamite).

Extra problems I recommend you work (not to be turned in)

- Lets consider the cross section for an electron scattering off of another electron. We will make the hand waving argument that the scattered electron will be deflected by a significant angle if the maximum potential energy it feels during the scattering is comparable or greater than the kinetic energy of the electron. So we might hypothesize a "scattering probability" which goes as

$$P(r) = \frac{1}{E_k} \cdot \frac{e^2}{4\pi\epsilon_0 r}.$$

With this assumption, what scattering cross section do you calculate for an electron with kinetic energy E_k? Surprising, isn't it?

- Actually do the integrals to calculate the correct critical mass. Note that this is the critical mass for pure ^{235}U, which is rather hard to make. If you use natural uranium, you have to take into account the lower density of ^{235}U nuclei and the absorption due to ^{238}U nuclei. This results in a much, much larger bomb.

- If we assume that our bomb has a reproduction factor of 1.2, how long does it take for the explosion to occur?