Physics 222 CID ____________

Homework 17

Score ________ / 30

1. (3 pts) Imagine that I have two curved mirrors that form an optical cavity. The mirrors and the air around them are at a temperature of 300K. (a) How many photons are there in a particular cavity mode which has a wavelength of 400 nm? (b) If I could measure *all* of the photons in the cavity which had a wavelength of 400 nm, I would actually measure more than your answer to part (a). Why?

In the next three problems we will derive Planck's radiation law (not the way Planck did it, but the right way)...

2. (4 pts) Assume that I have a cubic shaped optical cavity with sides of length L at a temperature T. If you solve the classical wave equation by separation of parts, you will find that the solutions are of the form $A\sin(k_x x)\sin(k_y y)\sin(k_z z)\cos(\omega t)$ where $\omega = 2\pi c/\lambda$ and $\lambda = 2\pi/k$. Here k is the "total" wavenumber, given by $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. (a) What are the allowed values of k_x, k_y, and k_z? (b) If I measure the energy of a photon in the cavity, what possible values could I find? Write them in terms of integers n_x, n_y, and n_z. Note that the zero-point energy is not important, since it is a property of the cavity, not of any individual photon.

3. (4 pts) Derive the density of states for the cavity in the problem above. Hint: Once again, this is not affected by zero-point energy.

4. (6 pts) (a) What is $n(\epsilon)d\epsilon$ for the cavity? (b) What is $n(\nu)d\nu$, the number of particles in the cavity with a frequency between ν and $\nu + d\nu$? (c) What is $u(\nu)d\nu$, the energy per unit volume contained in photons with frequencies between ν and $\nu + d\nu$? Ta-da, you have derived the Planck blackbody distribution!

5. (3 pts) (a) Explain the difference between the classical theory, Einstein's theory, and Debye's theory of the specific heat of a solid. (b) Which theory best fits experimental data?

6. (6 pts) For room temperature ($T = 300$K) the energy difference between the $n = 1$ and $n = 2$ states in hydrogen is very large compared to kT. As such, in a gas of room temperature hydrogen we would expect to find almost all of the atoms in the $n = 1$ state. For simplicity we will use Maxwell-Boltzmann statistics for this problem. (a) If I pick a particular atom in the gas and measure its quantum state, what is the probability of finding it in the $n = 3$, $l = 2$, $m_l = -1$ state? (b) What is the probability of finding the atom in the $n = 2$ shell? (c) If the gas were really hot, we couldn't assume that the almost all of the atoms will be in the $n = 1$ shell. This makes things harder. But it is still easy to find ratios of probabilities. For a gas at $T =$one million kelvin, what is the ratio of the probability of finding the electron in an atom in the $n = 3$ shell to the probability of finding it in the $n = 2$ shell? Note: you need remember to consider degeneracy due to the spin of the electron for this problem, but you don't have to worry about fine structure splitting since the splitting is so small.

7. (4 pts) (a) I shine laser light on a large sample of atoms with just enough intensity such that the rate of stimulated emission is equal to the spontaneous emission rate. After the laser has been on for a long time what fraction of the atoms are in the excited state? (b) Now I take the limit as laser power goes to infinity. Now what fraction of the atoms are in the excited state?

Extra problems I recommend you work (not to be turned in)

- I want to make a laser by pumping a two state system with a flash-lamp which happens to put out a large amount of light which is resonant with my two state system. Explain why this laser will never work, no matter how much light my flash-lamp puts out or how good the lasers mirrors are.
- In my lab we have a Ti:sapphire laser that we pump with a beam from an argon ion laser. If the threshold pump power is 2.45 Watts and the slope efficiency is 0.2 (or 20%), how much power will I get out of the Ti:sapphire laser if I pump it with 3 Watts of argon laser light? Assume that this pump power is small enough that we aren't anywhere close to saturating the inversion.

- What are some possible ways to make a laser run in a single mode of the optical cavity?
- Using equation 9.49 in your text, explain why it is harder to make ultraviolet lasers than infrared lasers.
- When an atom emits a photon via stimulated emission, the photon travels in the same direction as the photon that emitted it. But when it decays spontaneously, the emitted photon can go in just about any direction. Imagine that you shot a laser at a glass cell filled with mercury vapor, and that the laser's frequency is just right to drive mercury atoms from the ground state to an excited state. The atoms scatter some of the laser light, and you collect some of this scattered light with a lens and measure it with a photo diode. Using calculations similar to the ones you did in problem 7, sketch a plot of the measured level of scattered light as a function of the intensity of your laser beam.