

You may find the following integrals useful on this assignment:

$$\int_0^{\infty} x e^{-x} dx = 1 \qquad \int_0^{\infty} \sqrt{x} e^{-x} dx = \frac{\sqrt{\pi}}{2} \qquad \int_0^{\infty} x^{3/2} e^{-x} dx = \frac{3\sqrt{\pi}}{4}$$

1. (7 pts) Use Maxwell-Boltzmann statistics for this problem. (a) What is the density of states for a one-dimensional harmonic oscillator with oscillation frequency ω ? (b) What is $n(\epsilon)d\epsilon$ for N non-interacting particles in this potential? Give your answer in terms of ω , ϵ , N , T , and fundamental constants. (c) What is the total energy of all of the particles if the system is at temperature T ? (d) What is the average energy per particle if the system is at temperature T ?
2. (4 pts) Now let's consider quantum effects. (a) If I put N non-interacting spin 1 particles into a one-dimensional harmonic oscillator potential and cool the system down to 0 K, how many of the particles will be in the ground state? (b) How many of the particles will be in the ground state if the particles have a spin of 1/2?
3. (4 pts) What is the Fermi energy for N spin 1/2 particles in a 1-D harmonic oscillator with an oscillation frequency ω at absolute zero?
4. (7 pts) Find the density of states for a three dimensional infinite square well of size L . The energies will be given by $\epsilon = E_1(n_x^2 + n_y^2 + n_z^2)/3$. Give your answer in terms of E_1 , ϵ , and fundamental constants.
5. (8 pts) Use your results from the last problem to find the average energy per particle in a three-dimensional gas at a temperature T . Assume that the temperature is high enough that you can get away with using Maxwell-Boltzmann statistics.

Extra problems I recommend you work (not to be turned in)

- In our lab we trap atoms using lasers and magnetic fields. It turns out that the motion of the atoms in our trap is just the same as if they were in a conservative harmonic oscillator potential. Find the average total energy per particle for a gas of atoms trapped in a spherically symmetric harmonic oscillator potential whose potential is described by $U = Ar^2$.
- If you are good with numerical problems on a computer, plot how the Fermi energy changes with temperature for N particles in a one-dimensional infinite square well. Now do the same thing for a 3-D square well.
- Derive the Rayleigh-Jeans formula from scratch.