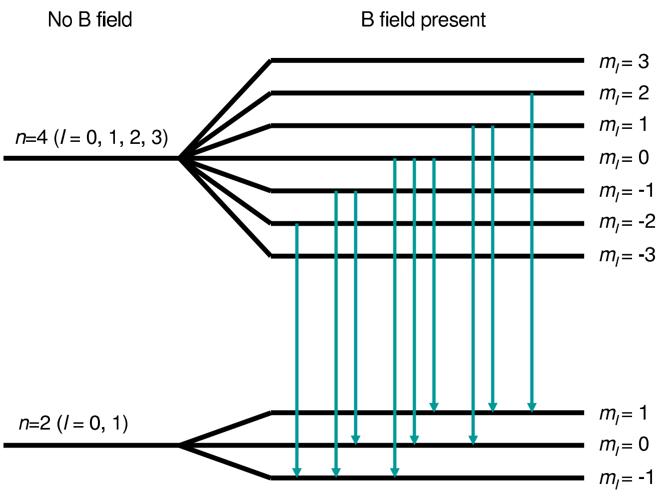
Physics 222 CID ____________

Homework 12

Score ________ / 30

1. (7 pts) Look at the diagram below which illustrates how the magnetic sublevels of the $n = 4$ and $n = 2$ levels split in energy when a magnetic field is applied according to the normal Zeeman effect. Now consider the spectral lines emitted when a hydrogen atoms makes a transition from the $n = 4$ to $n = 3$ energy level. (a) Make a drawing similar to the example below which shows how the $n = 4$ and $n = 3$ energy levels split in the presence of a magnetic field according to the normal Zeeman effect. Draw arrows from each $n = 4$ magnetic sublevel down to each $n = 3$ sublevel which can be reached through a dipole-allowed transition. (b) If I apply a magnetic field of strength B and only consider the normal Zeeman effect, how many individual lines does this single spectral line break into? (c) What spacing (in Hz) of the individual spectral lines does the normal Zeeman effect predict?



2. (4 pts) Lets pretend that electrons are little classical (i.e., not quantum mechanical or relativistic) spinning spheres of constant mass density with a radius of 10^{-15} meters (experiments have shown that the electron radius is at least this small). (a) At what angular velocity ω do the spheres need to spin for their angular momentum to be $\hbar\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}$? (b) What would the velocity of the outer edge of the sphere? The rotational inertia for a solid sphere is $I = \frac{2}{5}MR^2$.

3. (4 pts) (a) If a free electron is described by the quantum numbers $s = 1/2$ and $m_s = 1/2$, what possible results could I get if I measure the z component of the angular momentum of the electron? (b) After measuring the z component of the electron's angular momentum I measure the x component of its angular momentum. What possible results could I get? (c) After measuring the x component of the electron's angular momentum I measure the z component again. What possible values could I measure?

4. (4 pts) Explain in your own words why wave function symmetry results in the Pauli exclusion principle.

The following problems consider multiple non-interacting particles in a one-dimensional infinite square well. Using separation of variables you can find the allowed energies and wave functions for this system. (If the particles interact the problem is harder, because there is an extra term in Schrodinger's equation to account for the interaction energy.) The allowed energies turn out to be just $(n_1^2 + n_2^2 + ... + n_N^2)\pi^2\hbar^2/2mL^2$. There are N quantum numbers instead of one because there are N particles, and the total energy is just the sum of the energies of each particle. You can *almost* get the spatial part of the wave functions just by multiplying the wave functions of the individual particles. For example, if I have two particles in the well, the wave function is *almost* equal to

$$\psi(x_1, x_2) = \sqrt{\frac{2}{L}}\sin\left(\frac{n_1\pi x_1}{L}\right) \times \sqrt{\frac{2}{L}}\sin\left(\frac{n_2\pi x_2}{L}\right) = \frac{2}{L}\sin\left(\frac{n_1\pi x_1}{L}\right)\sin\left(\frac{n_2\pi x_2}{L}\right).$$

This isn't quite right, however, because it does not have the right symmetry.

5. (7 pts) Consider two non-interacting particles in a one-dimensional infinite square well of size L.

- **(a)** If the two non-interacting particles are neutrons (which have no charge, and have a spin quantum number $s = 1/2$), and both neutrons have the same m_s, what is the properly symmetrized spatial wave function $\psi(x_1, x_2)$ for the state which represents one particle in the $n = 1$ level and one in the $n = 2$ level?
- **(b)** Is it possible for us to put both neutrons into the $n = 1$ level of the infinite square well? Why or why not?
- **(c)** With that in mind, what is the properly symmetrized spatial wave function for the state which represents both particles in the $n = 1$ level?
- **(d)** If the two non-interacting particles are calcium-40 atoms (which have no net charge, and have a spin quantum number $s = 0$), what is the properly symmetrized spatial wave function for the ground state and first excited state of the system of two atoms?

6. (4 pts) Consider a system of 6 identical non-interacting particles in an infinite square well. What is the energy of the ground state of the system if the particles have a spin of (a) $s = 0$, (b) $s = 1/2$, (c) $s = 1$, and (d) $s = 3/2$?

Extra problems I recommend you work (not to be turned in)

- Use separation of variables to find the allowed energies for two identical particles in an infinite square well.
- Find the ground-state energy for 5 non-interacting particles in a harmonic oscillator potential if the particles have a spin quantum number of 0, 1/2, 1, 3/2, or 5/2.
- Find the ground-state energy for 6 non-interacting $s = 1/2$ particles in a three-dimensional infinite square well where each side is of length L.