

1. (4 pts) A hydrogen atom has the quantum numbers  $n = 3$  and  $l = 2$ . (a) If I measure the total energy of the electron, what possible values could I get? (b) If I measure the magnitude of the angular momentum, what possible values could I get? (c) If I measure the  $z$  component of angular momentum, what possible values could I get? (d) If I measure the  $x$  component of angular momentum, what possible values could I get? Give all answers in terms of the hydrogen atom ground state energy  $E_1 = -13.61\text{eV}$  and fundamental constants.
2. (6 pts) Find the total energy (in terms of  $E_1$ ), the magnitude of the total angular momentum  $L$ , and all of the possible values for the  $z$  component of angular momentum  $L_z$  for an electron in a hydrogen atom in the following states: (a) 1s, (b) 3p, (c) 3d.
3. (5 pts) In the last assignment I promised that we'd peek into Schrodinger's equation with a potential which was spherically symmetric (in other words, a potential which only depends on  $r$ ). This problem will make good on that promise. Start with equations 6.12-6.14 in the book. Now that we have Schrodinger's equation separated into three equations, how do we solve them? We saw in class that the  $\phi$  part was easy to solve, but we decided that we didn't have the proper tools to solve the  $\theta$  part. In fact, if you go on in physics and come to this problem again, you will find that the way it is solved is by manipulating it to look like another equation which you don't know how to solve, but that some guy named Legendre was able to solve using a set of polynomials which can be created using something called a generating function. So, you see, by sidestepping this problem, we aren't really doing much more hand-waving than will be done in higher division physics courses. So for now, we'll just note that (1) the  $\theta$  part can be solved, (2) some of the solutions are in your book and have been presented in class, so you kind of know what they look like, and (3) *they do not depend on the actual form of  $U(r)$ , as long as the potential is only a function of  $r$ !* With that stated, lets focus on just the  $r$  equation.
  - (a) For a square box, the solutions to the time-independent Schrodinger equation was easy. Now consider a spherical box — in which  $U(r) = 0$  as long as  $r < a$  and  $U(r) = \infty$  when  $r \geq a$ . Plug  $U(r) = 0$  into the equation 6.14 (the book has already inserted  $U(r) = -e^2/4\pi\epsilon_0 r$ , so simply replace that term with zero). After writing this equation down on your homework, do one of the two things to get full credit for this part of the problem: 1. Solve the equation, or 2. Write down, "Wow, even with that simple potential I can't solve the equation!" I suggest option 2. It isn't any easier if you plug in the  $1/r$  potential of a hydrogen atom. If you take Physics 451 and 452 you will probably see the hydrogen atom again, at which point you will solve the radial part in a manner similar to the unfulfilling one you will use for the  $\theta$  part.
  - (b) For the infinite spherical well, what are the boundary conditions on  $r$ ? Hint: there is one condition when  $r = a$ , and another when  $r = 0$ . To figure out the  $r = 0$  condition, consider what happens if I approach  $r = 0$  from the  $\phi = 0$  side, and then match that up with what happens when I approach from the  $\phi = \pi$  side. Then think about how kinks in the wave function result in infinite kinetic energy. (Note that for the hydrogen atom, the potential goes to  $-\infty$  at  $r = 0$ , so kinks aren't a problem.)
4. (4 pts) Look at figure 6.11 in the text. (a) In which of the following states is an electron most likely to be found near the nucleus, 1s, 2s, or 3s? (b) In certain types of nuclear reactions, an electron combines with a proton to make a neutron. In which of the following states is the electron most likely to be found "inside" the proton, 3s, 3p, or 3d?
5. (3 pts) A hydrogen atom is in the  $n = 3, l = 1, m_l = 1$  state. State whether the atom can make an electric dipole transition to the following states: (a)  $n = 4, l = 1, m_l = 1$ , (b)  $n = 4, l = 2, m_l = 1$ , (c)  $n = 12, l = 0, m_l = 0$ .
6. (4 pts) Consider the  $n = 3, l = 2, m_l = 2$  state of the Hydrogen atom. (a) If I stop time and move around a circle in the  $xy$  plane centered on the origin, how many complete oscillations do I see in the wave function as I go from  $\phi = 0$  to  $\phi = 2\pi$ ? (b) If I walk along the  $x$  axis from a point just barely to the  $+x$  side of the origin out almost to infinity (I don't quite get there though), how many times do I see the wave function cross through zero?

7. (4 pts) In your own words, explain why equation 6.34 in the text must be non-zero in order for the transition from  $\psi_n$  to  $\psi_m$  to be dipole allowed.

**Extra problems I recommend you work (not to be turned in)**

- Show that if  $U(r) = ar^2$  you *can* solve the time-independent Schrodinger's equation. This is done by converting your potential into Cartesian coordinates and solving in terms of  $x$ ,  $y$ , and  $z$ . When you separate variables, you get three simple harmonic oscillator equations, which we know the solutions to.
- Start with the time-independent Schrodinger equation in spherical coordinates. You can find this in equation 6.3 in your book. This equation involves three variables,  $r$ ,  $\theta$ , and  $\phi$ . Let's try to separate it into three equations which each involve only one variable.
  - (a) Assume that  $\psi$  can be factored into three parts, each depending on only one variable:  $\psi(r, \theta, \phi) = R(r) \cdot \Theta(\theta) \cdot \Phi(\phi)$ . Plug this into the equation, and then manipulate the equation until only one term in the equation contains anything which depends on  $\phi$ , and such that this term does not depend on  $r$  and  $\theta$ . Hint: the term should look like  $(1/\Phi)\partial^2\Phi/\partial\phi^2$ .
  - (b) Now note that since this term only depends on  $\phi$ , it doesn't change when  $r$  or  $\theta$  change. As such, the sum of the other terms must not change when  $r$  or  $\theta$  change. Also, since the other terms don't contain  $\phi$ , they don't change when  $\phi$  changes. So to make the equation true for all values of  $\phi$ , the term with  $\phi$  in it must not change with  $\phi$ — it must be a constant. Let's define the constant to be  $-m_l^2$ . In other words, let's define:

$$\frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial \phi^2} = -m_l^2.$$

Use this definition to separate the equation into two separate equations, one which only involves  $\phi$ , and one which involves  $r$  and  $\theta$ . Hint: one of the equations should look like equation 6.12 in the text.

- (c) Now manipulate the equation involving  $r$  and  $\theta$  such that there are terms that depend on  $r$  and terms which depend on  $\theta$  but no terms that depend on both. Set the  $r$  terms equal to  $l(l+1)$  and separate this equation to get equations 6.13 and 6.14 in the textbook.
- (a) Show that by adding  $(1/\sqrt{2})(\psi_{n,l,m} \pm \psi_{n,l,-m})$  you get energy eigenstates which are real, and which have real nodes in  $\phi$  (actually going to zero, not just oscillating around in the complex plane). These are the energy eigenstates which you typically see in chemistry textbooks. (b) Explain why these functions are eigenstates of  $E$  and  $L$ , but not of  $L_z$ .