

- (6 pts) (a) What three measurable quantities do the three quantum numbers of a hydrogen atom ( $n$ ,  $l$ , and  $m$ ) represent? (b) If  $n = 4$ , what values can  $l$  have? (c) If  $l = 3$ , what values can  $m_l$  have? (You need to peek ahead to section 6.6 if you haven't already read it to find out what  $m$  represents).
- (6 pts) Find the principle quantum number  $n$ , the angular momentum (or orbital) quantum number  $l$ , and all of the possible values for the magnetic quantum number  $m_l$  for an electron in a hydrogen atom in the following states: (a) 1s, (b) 3p, (c) 3d.

The hydrogen atom is our first introduction to 3D problems. Some properties of the hydrogen atom (like the fact that it has three quantum numbers) are simply due to the fact that is a 3D problem. Others are due to the fact that the potential energy only depends on  $r$ , and not on  $\theta$  or  $\phi$ . To get a better feel for how this all comes together, we will take a look at two other interesting potentials in this and the following homework set. We'll do a 3D infinite square well on this assignment. Like the hydrogen atom, this problem is three dimensional. The potential, however, is a function of  $x$ ,  $y$ , and  $z$ , not just a function of  $r$ . On the next assignment we'll go over a potential which is just a function of  $r$ .

- (10 pts) Consider a particle in a three-dimensional infinite square well potential. This approximates, for example, an electron trapped in a tiny cube of metal. The particle is contained inside a hard-walled cube, each side of the cube having a length  $L$ . Inside the cube  $U = 0$ , and outside the cube  $U = \infty$ . (a) Write down the time-independent Schrodinger equation inside the box (where  $U(x, y, z) = 0$ ) in Cartesian coordinates. (b) Assume that there are solutions to this equation which have the form  $\psi(x, y) = F(x) \cdot G(y) \cdot H(z)$ . Plug this into the equation you wrote down in part (a) above, and show that the equation can be written in the form

$$E = A(x) + B(y) + C(z)$$

where  $A(x)$  is something that only depends on  $x$ ,  $B(y)$  is something that only depends on  $y$ , and  $C(z)$  is something that only depends on  $z$ . (c) Set the pieces equal to constants to create three independent differential equations. Solve them and apply the boundary conditions ( $\psi = 0$  when  $x < 0$ ,  $y < 0$ ,  $z < 0$ ,  $x > L$ ,  $y > L$ , or  $z > L$ ) to find the wave functions and energies of the energy eigenstates for this potential. Write these solutions as a function of  $n_x$ ,  $n_y$ , and  $n_z$ , three positive integers (the three quantum numbers of the particle). Remember to normalize the wave functions.

- (8 pts) It happens that there are other solutions to the time-independent Schrodinger equation in problem 4 which are not of the form  $\psi = F(x) \cdot G(y) \cdot H(z)$ . But the solutions which you found constitute a complete basis set — any solutions *not* of this form can be written as a sum of solutions which *are* of this form. Unlike the time-dependent Schrodinger equation, the time-independent equation is not linear, and any arbitrary sum of solutions is not itself necessarily a solution to the equation. (a) Since the solutions to the time-independent Schrodinger equation are energy eigenstates, it makes sense to think that if we add solutions with different energies, that the result will not be a solution to the equation. Show that adding the  $n_x = 1$ ,  $n_y = 1$ ,  $n_z = 1$  solution to the  $n_x = 1$ ,  $n_y = 2$ ,  $n_z = 1$  solution does not result in a solution to the time-independent Schrodinger equation for  $U = 0$ . (b) Show that the sum of the  $n_x = 1$ ,  $n_y = 2$ ,  $n_z = 1$  solution and the  $n_x = 2$ ,  $n_y = 1$ ,  $n_z = 1$  solution IS a solution to the time-independent Schrodinger equation. In general, if two states are degenerate (meaning that they have the same energy), they can be added together to make another energy eigenstate which will solve the time-independent Schrodinger equation<sup>1</sup>.

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<sup>1</sup>Consider the classical problem of a pendulum free to swing in two directions. You can have a solution where it only swings in  $x$  and a solution where it only swings in  $y$ , or a sum of those two solutions. If you add those two solutions together with the same amplitude, you get a solution in which the pendulum goes in circles. If you subtract them, you get a solution which goes in circles in the opposite direction. All possible solutions can be written as a sum of the  $x$  and  $y$  solutions. But they can also be written as a sum of the clockwise and counter-clockwise solutions. So  $x$  and  $y$  form a complete "basis set" of solutions. But since the two solutions are degenerate, I can combine them to make two different states which form an equally good basis set.

**Extra problems I recommend you work (not to be turned in)**

- Find the wave functions and energies of the energy eigenstates for a particle in a three dimensional infinite square well with dimensions  $L_x$ ,  $L_y$ , and  $L_z$ . Discuss how making the well have different dimensions in  $x$ ,  $y$ , and  $z$  affects degeneracy.