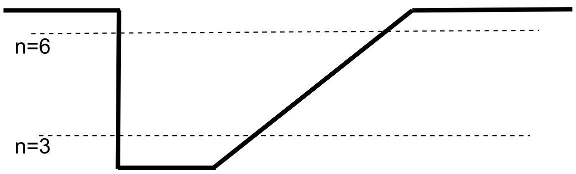
Physics 222 CID ____________

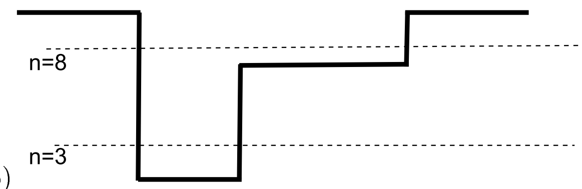
Homework 9

Score ________ / 30

1. (4 pts) The following plots show the potential U in dark lines. The energies of several stationary states are shown with the dotted lines. Using the dotted lines as the x axis for each of these states, sketch what the wave functions of these states will look like on the figures below. Assume that the ground state is labeled as $n = 1$.



(a)



(b)

2. (6 pts) The work function for aluminum is 4.08 eV. This means that if an electron in free space has a potential energy equal to zero, an electron in aluminum has a potential energy equal to -4.08 eV. Consider an electron with a very well defined kinetic energy of 5.11 eV which is traveling along, minding its own business, inside of a piece of aluminum. Suddenly it's wave packet reaches the edge of the aluminum. Since the electron hits it at normal incidence, we can use our one dimensional step potential equations. (a) What is the momentum p and the wavenumber k of the electron when it is in the aluminum? (b) What will its momentum and wavenumber be if it leaves the aluminum? (c) Some time after the wave function collides with the surface we measure the position of the electron. What is the probability that we will find that it reflected back into the aluminum?

3. (2 pts) Two pieces of Aluminum are pressed together, but a layer of oxide and contamination on one of the pieces prevents the aluminum pieces from actually touching on a microscopic scale. The junk has a work function of 2.11 eV and is 1.1 nm thick. What are the odds that an electron which has a kinetic energy of 1 eV will tunnel through the layer an into the other piece of aluminum.

4. (2 pts) Imagine that an electron is trapped in a nanoparticle. We don't know enough about the particle to calculate the potential that the electron feels, so we hope that we can at least approximate it with a 1-D simple harmonic oscillator for low n states. I do an experiment and find that the difference in energy between the ground state ($n = 0$) and first excited ($n = 1$) state is equal to $E_1 - E_0 = 10.2$ eV. If a simple harmonic oscillator is a good approximation, what should the difference in energy between the $n = 1$ and $n = 2$ states be?

5. (3 pts) Consider a particle in a 1-D harmonic potential with a zero-point energy of 1.14 eV. In class we briefly discussed "selection rules" for absorbing and emitting photons. For a simple harmonic oscillator, "dipole allowed" transitions only occur between adjacent states — ones for which $n_{upper} = n_{lower} + 1$. (a) Taking these selection rules into account, what is the longest wavelengths of light that our particle is "allowed" to emit while confined to this potential well? (b) What is the shortest wavelength of light it is "allowed" to emit?

For the following problems, consider a particle in a harmonic potential which has a zero-point energy E_0. Give all answers in terms of fundamental constants, the energy-eigenstate wave functions for the harmonic oscillator ψ_n (the time-independent functions), E_0, the mass of the particle m, and ϕ (defined below). The integrals that you need are:

$$\int_{-\infty}^{\infty} \psi_n^* \psi_m dx = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \psi_n^* x \psi_m dx = \begin{cases} \sqrt{\frac{\hbar^2}{2mE_0} \cdot \frac{n+1}{2}} & m = n+1 \\ \sqrt{\frac{\hbar^2}{2mE_0} \cdot \frac{n}{2}} & m = n-1 \\ 0 & \text{otherwise} \end{cases}$$

where ψ_n represents the time-independent energy-eigenstate wave functions for a harmonic oscillator with a zero-point energy E_0.

6. (3 pts) (a) What is the energy of the ground state ($n = 0$)? (b) What is the energy of the three lowest energy states above the ground state ($n = 1$, 2, and 3)?

7. (4 pts) Assume that the wave function for the particle at time $t = 0$ is given by

$$\Psi(x, t = 0) = \sqrt{\frac{1}{3}}\psi_0(x) + e^{i\phi}\sqrt{\frac{2}{3}}\psi_1(x)$$

where ϕ is a constant and $\psi_0(x)$ and $\psi_1(x)$ are the ground and first excited state wave functions for the harmonic potential. (a) What is the wave function of the particle at some arbitrary time t? (b) What is $P(x)$ at some arbitrary time t? Express $P(x)$ in such a way that it is obvious that it is not complex.

8. (4 pts) (a) What is the expectation value $< E >$ for the particle described in the above problem at some arbitrary time t? (b) What is $< x >$ at some arbitrary time t. Make sure to write them in a way that it is obvious that they are not complex.

9. (3 pts) What is the earliest non-zero time at which $P(x)$ for the particle described above will be the same as it was at time zero? (This is known as a "recurrence.")

<u>Extra problems I recommend you work (not to be turned in)</u>

- Now assume that a particle in the well described in problems 6-9 has a wave function at time $t = 0$ which is given by

$$\Psi(x, t = 0) = \sqrt{\frac{1}{5}}\psi_0(x) + e^{i\phi}\sqrt{\frac{2}{5}}\psi_1(x) + i\sqrt{\frac{2}{5}}\psi_5(x).$$

  (a) What is the earliest non-zero time at which $P(x)$ will be the same as it was at time zero? Hint: you don't need to actually find $P(x)$ to answer this. Just think about what the relative phases of the three pieces will be. (b) If I measure the energy of the particle, what possible outcomes can I get? (c) What is $< E >$ for this particle? (d) What is $< U >$ for this particle? (e) What is $< KE >$?

- When using the "standard deviation" definition of uncertainty (which is what we almost always use), you can find the uncertainty in an observable q using the equation $\Delta q = \sqrt{< q^2 > - < q >^2}$. Use this to find Δx and Δp for the ground state of an infinite square well, and show that they obey the position-momentum uncertainty relation. The integral you will need is

$$\int_0^{\pi} x^2 \sin^2(x) dx = \frac{\pi^3}{6} - \frac{\pi}{4}$$

- Draw some potentials and practice sketching what the stationary state wave functions will look like.

- Find the phase shift when a neutron with a kinetic energy of 20 eV reflects off of a 40 eV potential step.

- What is the probability of finding the neutron in the "classically forbidden" area.

- For for a 1-D infinite square well the rule is that dipole allowed transitions only occur between one even and one odd n state. For example, if the electron were in the $n = 4$ state, it would be "allowed" to absorb light and go to the $n = 5$, 7, 9, ... state, etc., or to emit light and drop to the $n = 1$ or $n = 3$ state. If it were in the $n = 5$ state, it would be allowed to absorb light and go to the $n = 6$, 8, 10, ... state, etc., or to emit and drop to the $n = 2$ or $n = 4$ state. Taking this into account, what are the longest three wavelengths that an electron could emit if it were trapped in an infinite square well with a zero-point energy of 2.45 eV?

- Imagine some arbitrary wave function $\Psi_1(x, t)$, who's exact form I will not reveal. Now imagine a second wave function which is just the first one multiplied by a constant phase factor: $\Psi_2(x, t) = e^{i\phi}\Psi_1(x, t)$ (where ϕ is a constant). (a) Show that $P_1(x, t) = P_2(x, t)$. (b) Show that $< x >$ is the same for both wave functions. (c) Show that $< p >$ is the same for both wave functions. This is evidence of a more general principle; if I multiply every wave function in a system by a constant phase factor, it will not have any change on any measurable quantity.