Physics 222 CID ____________

Homework 8

Score ________ / 30

You might find the following integral useful on this assignment. Here n is an integer.

$$\int_0^{n\pi} x\sin^2(x)dx = \frac{n^2\pi^2}{4}$$

1. (3 pts) Lets look at some operators acting on the function $f(x) = x^4$. Calculate $\hat{x}\hat{p}f(x)$ and $\hat{p}\hat{x}f(x)$ and show that they are not the same (i.e., $\hat{x}$ and $\hat{p}$ do not commute).

2. (3 pts) (a) Use equation 5.46 and the information in section 5.7 to find the full time-dependent wave functions of the n^{th} energy eigenstate of a particle in a one-dimensional box Ψ_n as a function of n, x, t, L (the size of the box), E (the energy of the eigenstate), and fundamental constants. (b) Show that $P(x,t)dx$ for these energy eigenstates does not change in time.

3. (5 pts) Use the full time-dependent waveforms to find the expectation values $< x >$, $< p >$, $< p^2 >$, $< E >$, and $< KE >$ for the nth state of an infinite square well as a function of time. Remember that

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \quad \text{and} \quad \hat{KE} = \frac{\hat{p}^2}{2m}.$$

Also remember that when an operator is squared, that means that you operate twice. (Note that time should cancel out in all of these expectation values. Energy eigenstates — also known as stationary states — have this unique property that although the wave function evolves in time, it does it in such a simple way that no physically measurable property changes in time.)

4. (4 pts) A pure momentum state for a free particle has a wave function of the form:

$$\Psi = Ae^{i(kx-\omega t)}.$$

These states are not normalizable, so they can't describe a real particle. But they are still very useful. Often we will describe a real particle as a sum of these states (using a complex Fourier transform). So it's worth thinking about them a bit. (a) Use the criterion in equation 5.34 to prove that a wave function of this form is an are eigenstates of $\hat{E}$, $\hat{p}$, and $\hat{KE}$. (b) Show that it is not an eigenstates of or $\hat{x}$.

5. (5 pts) Imagine an electron traveling in free space with a wave function given by $\Psi = A\left[e^{i(k_1x-\omega_1t)} + e^{i(k_2x-\omega_2t)}\right]$ where A, k_1, k_2, ω_1, and ω_2 are constants. (a) What is the probability function $P(x,t)$? Write it in such a way that it is obvious that it is real (not complex). (b) What are the expectation values $< k >$, $< \omega >$, $< p >$ and $< E >$? (Hint — you don't have to do an integral, but you can if you want to in order to check your intuition).

For the next few problems we will consider a particle in an infinite square well of length L which has been prepared in such a way that its wave function is a superposition of two energy eigenstates. At time $t = 0$ the wave function inside the well is equal to

$$\Psi(x, t=0) = A\left[\sin\left(\frac{\pi x}{L}\right) + i\sin\left(\frac{2\pi x}{L}\right)\right]$$

where A is a real constant.

6. (4 pts) (a) What is A? (b) If I hadn't told you that A was real (i.e., if it could be complex), what other values could it have? (c) If we let A be complex, how will that change $P(x)$?

7. (3 pts) What is the wave function equal to at some arbitrary time t?

8. (3 pts) What is $< E >$at some moment in time t? If you think about this for a moment, you might find an argument which lets you find $< E >$ without doing an integral.

Extra problems I recommend you work (not to be turned in)

- If we measured the location of the particle in problems 5 through 7 at time $t = 0$, what would be the probability of finding it between $x = 0$ and $x = L/2$?
- How much could your answer to the above have changed if we had allowed A to be complex?
- If we measure the location of the particle at time $t = mL^2/2\pi\hbar$, what would be the probability of finding it between $x = 0$ and $x = L/2$?
- At what moments in time is $P(x)dx$ exactly the same as it is at time $t = 0$. This is known as a recurrence.